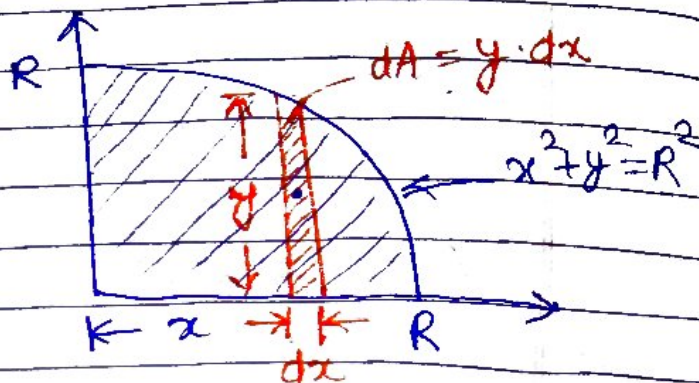


Que: Find the centroid of shaded area.

$$\bar{x} = \int \bar{x} \frac{dA}{A}$$

$$\bar{y} = \int \bar{y} \frac{dA}{A}$$



~~$$\bar{y} = \int \bar{y} \cdot d$$~~

$$\bar{y} = \frac{y}{2}$$

$$dA = y \cdot dx$$

$$\bar{y} = \int \bar{y} \cdot \frac{dA}{A} = \int \frac{y}{2} \cdot \frac{y dx}{A} = \int \frac{y^2 dx}{2A}$$

$$x^2 + y^2 = R^2$$

$$y^2 = (R^2 - x^2)$$

$$= \frac{1}{2A} \int (R^2 - x^2) dx$$

$$= \frac{1}{2A} \left[\int R^2 dx - \int x^2 dx \right]$$

$$= \frac{1}{2A} \left[R^2 \cdot x - \frac{x^3}{3} \right]_0^R$$

$$= \frac{1}{2 \cdot \frac{1}{4} \pi R^2} \left[R^3 - \frac{R^3}{3} \right]$$

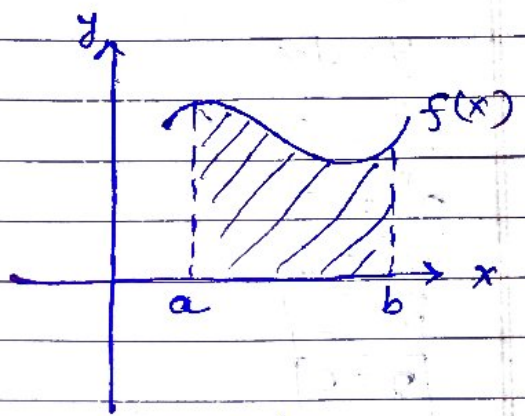
$$= \frac{2}{\pi R^2} \cdot \left[\frac{2R^3}{3} \right] = \boxed{\frac{4R}{3\pi}}$$

Same for \bar{x}

$$\bar{x} = \bar{y} = \frac{4R}{3\pi}$$

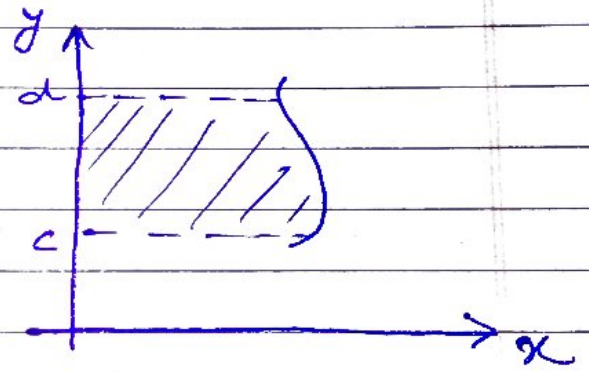
Area under any curve at x-axis :-

$$\text{Area} = \int_a^b f(x) dx$$



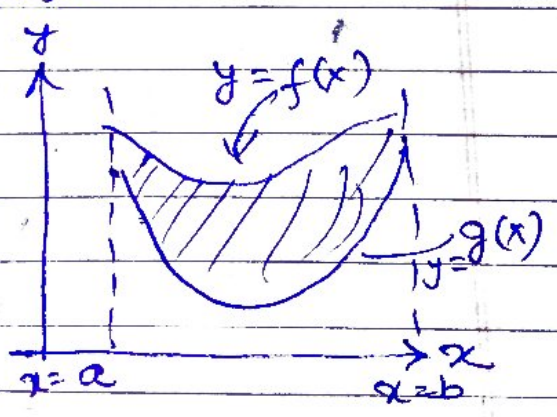
Area under any curve at y-axis

$$\text{Area} = \int_c^d f(y) dy$$



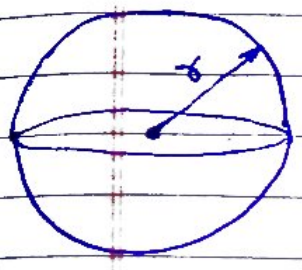
Area between the two curves :-

$$\text{Area} = \int_a^b [f(x) - g(x)] dx$$



Volume of Shapes

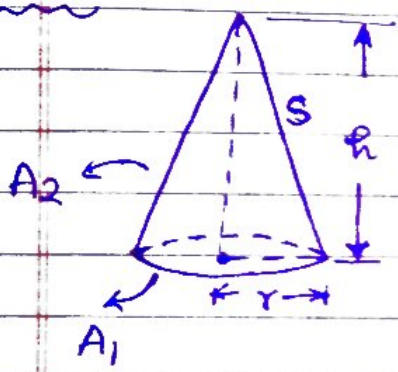
① Sphere



$$\text{Surface Area} = 4\pi r^2$$

$$\text{Volume} = \frac{4}{3}\pi r^3$$

② Cone



$$S.A = \pi r s + \pi r^2$$

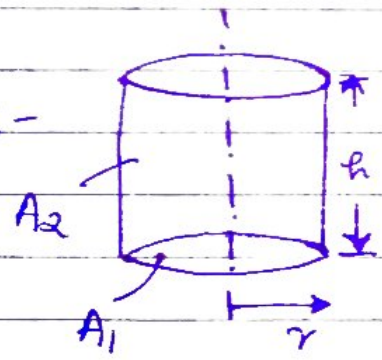
$$V = \frac{1}{3}\pi r^2 h$$

$$s = \sqrt{r^2 + h^2}$$

$$\text{Area } (A_1) = \pi r^2$$

$$A_2 = \pi r s$$

③ Cylinder



$$A_1 = \pi r^2$$

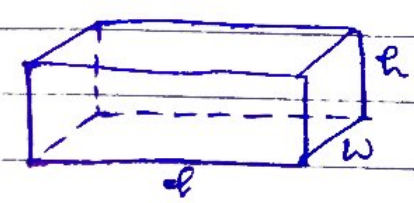
$$A_2 = 2\pi r h$$

$$T.S.A = 2A_1 + A_2$$

$$= 2\pi r^2 + 2\pi r h$$

$$\text{Volume} = \pi r^2 h$$

④ Rectangular Prism



$$SA = 2(lw + lh + wh)$$

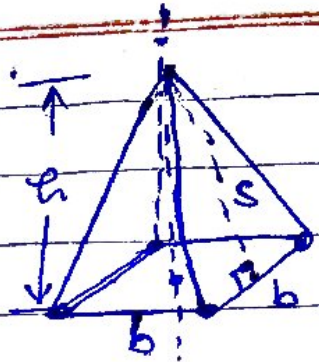
$$V = lwh$$

⑤

Pyramid

$$A = 2bs + b^2$$

$$V = \frac{1}{3} b^2 h$$

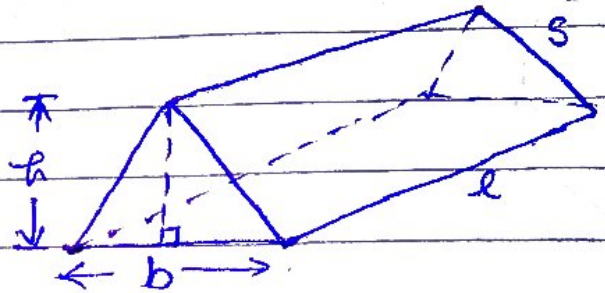


⑥

Prism:-

$$SA = bh + 2ls + lb$$

$$V = \frac{1}{2} bhl$$



Moment of Inertia of Plane area :-

- MOI is the name given to rotational inertia, the rotational analog of mass of linear motion.
- It appears in the relationships for the dynamics of rotational motion.
- MOI must be specified with respect to the chosen axis of rotation.

Rotational - Linear Relationships.

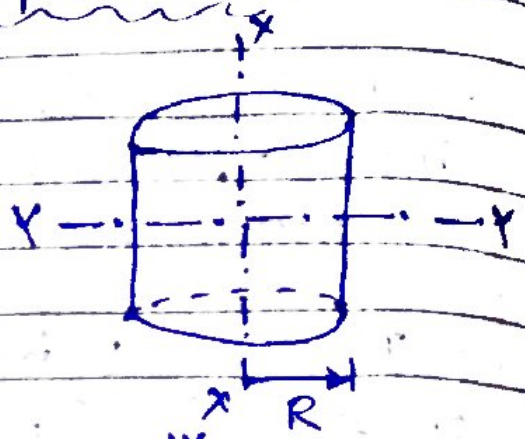
	Linear motion	Rotational Motion
Position	x	θ
velocity	v	ω
Acceleration	a	α
Motion eq ⁿ	$x = vt$ $v = u + at$ $x = ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2as$	$\theta = \omega t$ $\omega = \omega_0 + \alpha t$ $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ $\omega^2 = \omega_0^2 + 2\alpha\theta$
mass	m	I
Newton's II nd law	$F = ma$	$\tau = I \cdot \alpha$
Momentum	$p = mv$	angular momentum $L = I\omega$
work	$W = F \cdot d$	$W = \tau \cdot \theta$
Kinetic energy	$KE = \frac{1}{2}mv^2$	$KE = \frac{1}{2}I\omega^2$
Power	$P = Fv$	$P = \tau\omega$

Some common Moments of Inertia

① Solid cylinder

$$I_{xx} = \frac{1}{2} MR^2$$

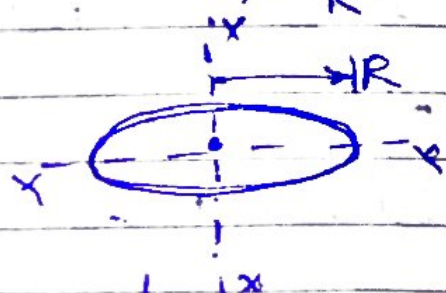
$$I_{yy} = \frac{1}{4} MR^2 + \frac{1}{12} ML^2$$



② Ring

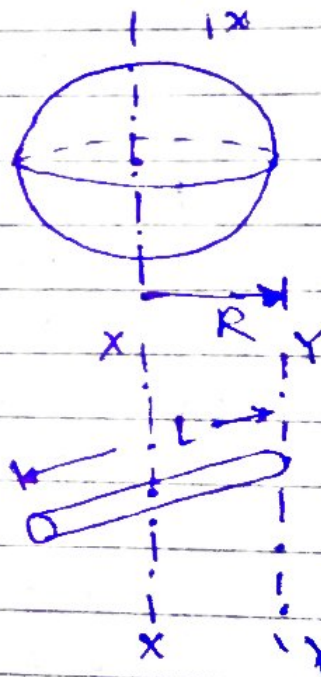
$$I_{xx} = MR^2$$

$$I_{yy} = \frac{1}{2} MR^2$$



③ Solid sphere

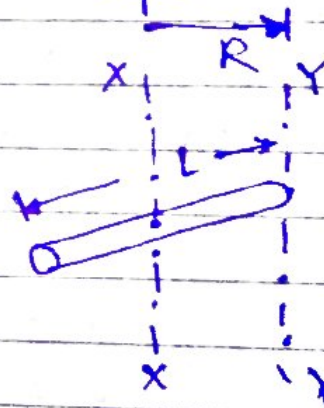
$$I = \frac{2}{5} MR^2$$



④ Rod about center

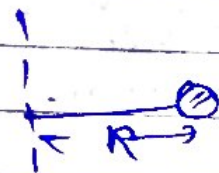
$$I_{xx} = \frac{1}{12} ML^2$$

$$I_{yy} = \frac{1}{3} ML^2$$



⑤ Point mass

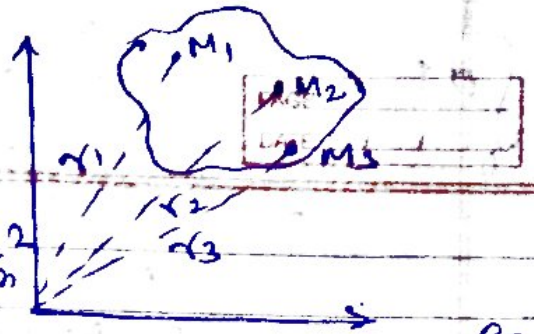
$$I = MR^2$$



General Form:

$$I = \sum_{i=1}^n m_i r_i^2$$

$$= m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2$$



$$\rho = \frac{m}{V}$$

$$m = \rho \cdot V$$

Que: mass MOI of cone?

let mass = M
density = ρ

$$dm = \rho \pi r^2 \cdot dz \quad \text{--- (1)}$$

$$\rho = \frac{M}{V}$$

$$= \frac{M}{\frac{1}{3} \pi R^2 H}$$

$$\rho = \frac{3M}{\pi R^2 H} \quad \text{--- (2)}$$

$$dm = \frac{3M}{\pi R^2 H} \cdot \pi r^2 dz = \frac{3M}{R^2 H} r^2 dz$$

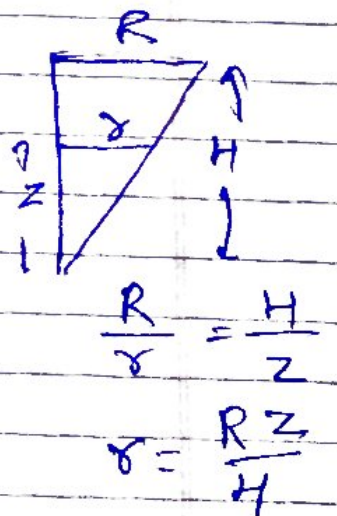
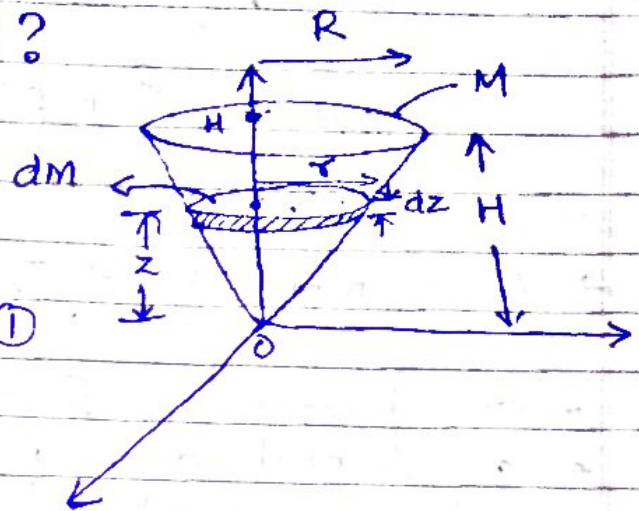
$$dm = \frac{3M}{H} \frac{r^2}{R^2} dz$$

$$dI = \frac{1}{2} dm r^2$$

$$= \frac{1}{2} \frac{3M}{H} \frac{r^2}{R^2} dz r^2$$

$$= \frac{1}{2} \frac{3M}{R^2 H} r^4 dz$$

$$= \frac{1}{2} \frac{3M}{R^2 H} \frac{R^4 z^4}{H^4} dz = \frac{3}{2} \frac{M R^2}{R^2 H^5} z^4 dz$$



$$\int_0^H dI = \int_0^H \frac{3}{2} \frac{M R^2}{R^2 H^5} z^4 dz$$

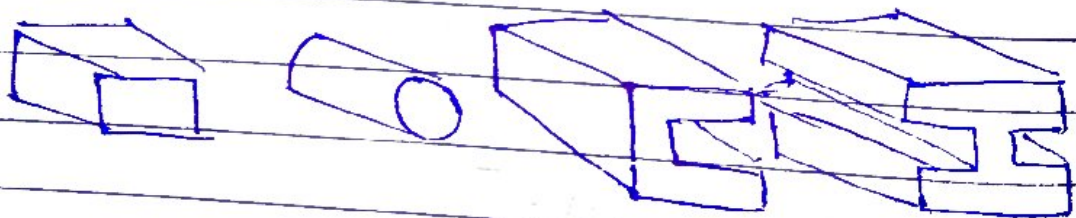
$$= \frac{3}{2} \frac{M R^2}{R^2 H^5} \left[\frac{z^5}{5} \right]_0^H$$

$$= \frac{3}{2} \frac{M R^2}{R^2 H^5} \frac{H^5}{5} = \frac{3}{10} \frac{M R^2}{R^2}$$

$$I = \frac{3}{10} M R^2$$

Area Moment of Inertia :-

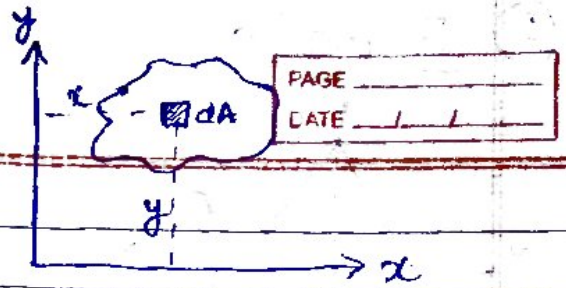
- also known as Second moment of area:
→ or Second area moment.
- is a geometrical property of an area.
- it reflects how its points are distributed with regards to an arbitrary axis.
- Denoted by I or J.
- ~~It is for axis perpendicular to the plane.~~
- It is distribution of area wrt to axis.
- Used in Beams.



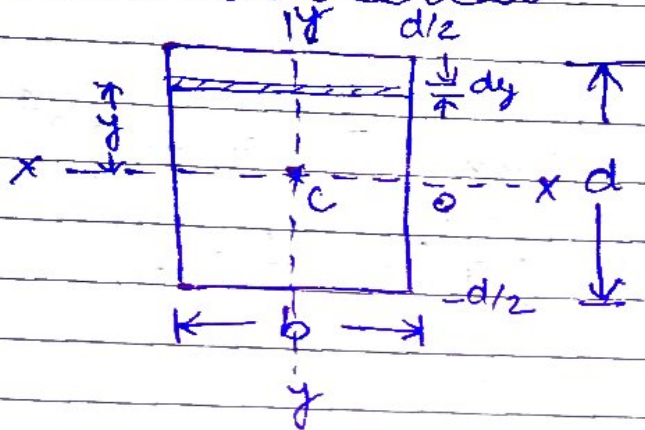
Expression :-

$$I_{xx} = \int y^2 dA = \iint y^2 dx dy$$

$$I_{yy} = \int x^2 dA = \iint x^2 dx dy$$



① Rectangular Section :-



X_{xx} = centroidal axis.

$$I_{xx} = \int y^2 dA$$

$$= \int y^2 b dy$$

$$= 2 \int_0^{d/2} y^2 b dy$$

$$= 2b \left[\frac{y^3}{3} \right]_0^{d/2}$$

$$= \frac{2}{3} b \frac{d^3}{8}$$

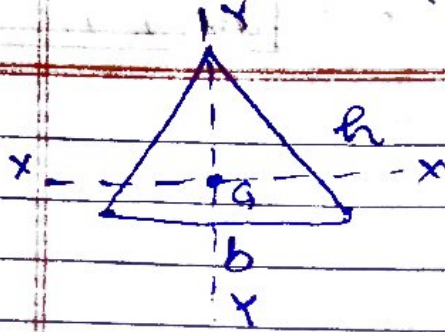
$$I_{xx} = \frac{bd^3}{12}$$

Similarly; $I_{yy} = \frac{db^3}{12}$ (mm^4) unit

$$\text{Polar MOI} = (I_c) = \frac{1}{12} (b^2 + h^2) bh$$

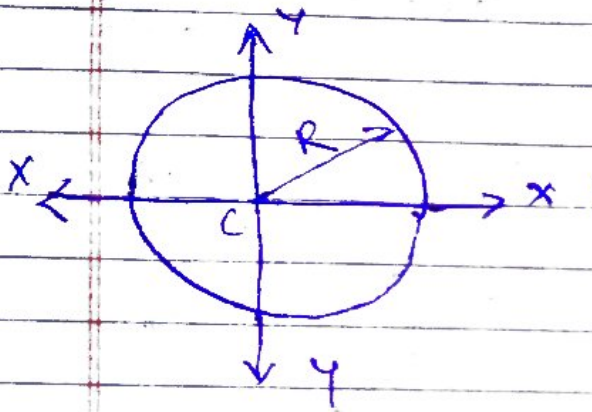
$$I_c = I_{xx} + I_{yy}$$

② Triangle



$$I_{xx} = \frac{bh^3}{36}$$

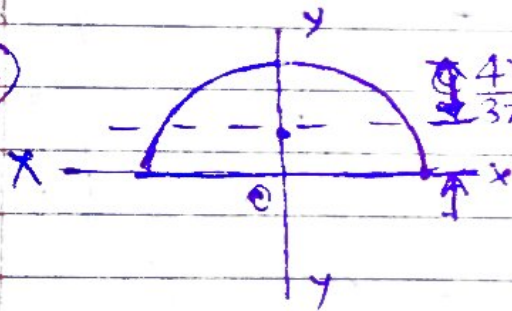
③ Circle



$$I_{xx} = I_{yy} = \frac{1}{4} \pi R^4$$

Polar (I_0) = $\frac{1}{2} \pi R^4$

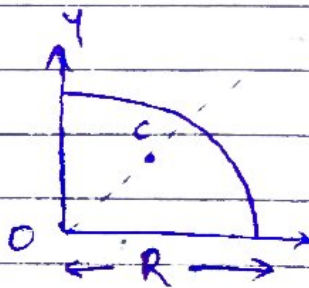
④



$$I_{yy} = I_{xx} = \frac{1}{8} \pi R^4$$

Polar (I_0) = $\frac{1}{4} \pi R^4$

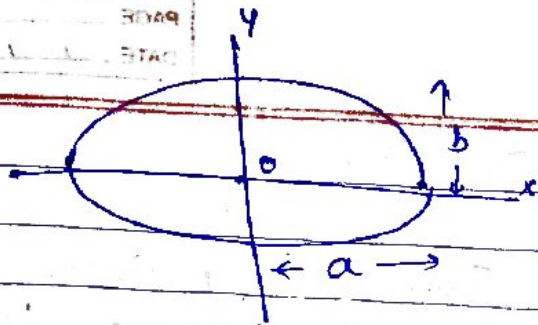
⑤



$$I_x = I_y = \frac{1}{16} \pi R^4$$

Polar (I_0) = $\frac{1}{8} \pi R^4$

6



$$I_x = \frac{1}{4} \pi a b^3$$

$$I_y = \frac{1}{4} \pi a^3 b$$

$$J_0 = \frac{1}{4} \pi a b (a^2 + b^2)$$

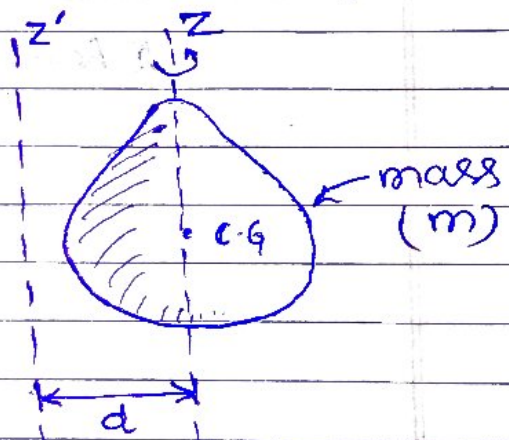
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Parallel axis Theorem :-

- also known as Huygen's - Steiner Theorem
- can be used to determine the mass MOI or Second MOA of any rigid body

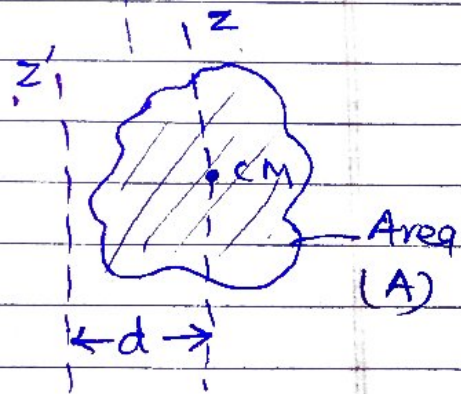
For Mass moment of Inertia :-

$$I_{z'} = I_z + m d^2$$



For Second MOA

$$I_{z'} = I_z + A d^2$$

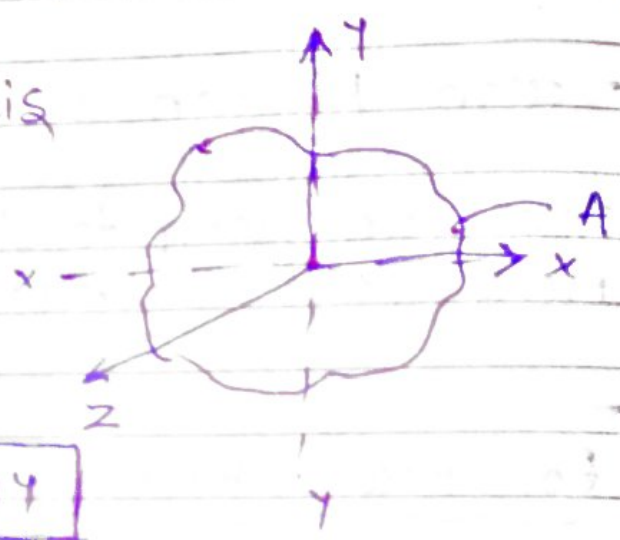


Perpendicular axis Theorem:-

It states that, "If I_{xx} & I_{yy} are the MOI of a given plane figure then there is also MOI which is \perp to the plane figure and both I_{xx} & I_{yy} .

→ also called Polar axis theorem. and

→ and MOI about z axis



$$I_{zz} = I_{xx} + I_{yy}$$

When $I_{zz} \rightarrow$ Polar MOI.