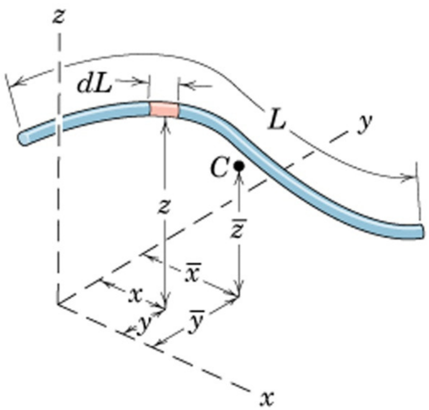
$$\bar{x} = \frac{\int x\,dm}{m} \quad \bar{y} = \frac{\int y\,dm}{m} \quad \bar{z} = \frac{\int z\,dm}{m}$$

Center of Mass and Centroids

Centroids of Lines, Areas, and Volumes

Centroid is a geometrical property of a body

→ When density of a body is uniform throughout, centroid and CM coincide



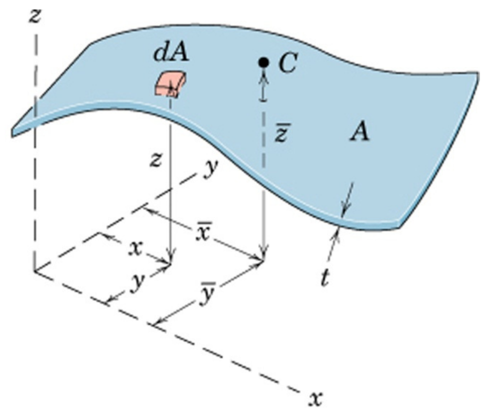
Lines: Slender rod, Wire

Cross-sectional area = A

ρ and A are constant over L

$dm = \rho A dL$; Centroid = CM

$$\bar{x} = \frac{\int x\,dL}{L} \quad \bar{y} = \frac{\int y\,dL}{L} \quad \bar{z} = \frac{\int z\,dL}{L}$$



Areas: Body with small but constant thickness t

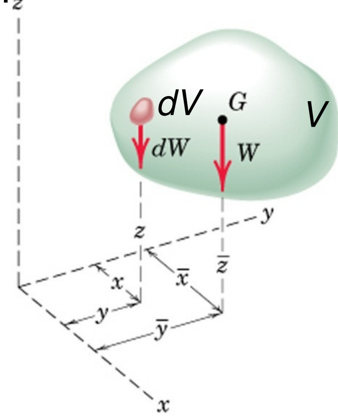
Cross-sectional area = A

ρ and A are constant over A

$dm = \rho t dA$; Centroid = CM

$$\bar{x} = \frac{\int x\,dA}{A} \quad \bar{y} = \frac{\int y\,dA}{A} \quad \bar{z} = \frac{\int z\,dA}{A}$$

Numerator = First moments of Area



Volumes: Body with volume V

ρ constant over V

$dm = \rho dV$ Centroid = CM

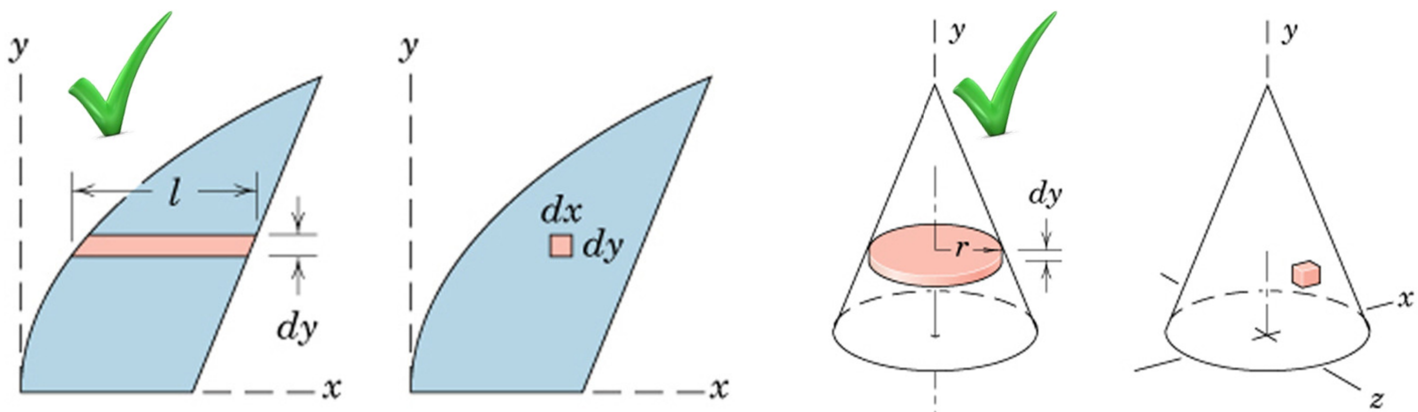
$$\bar{x} = \frac{\int x\,dV}{V} \quad \bar{y} = \frac{\int y\,dV}{V} \quad \bar{z} = \frac{\int z\,dV}{V}$$

Center of Mass and Centroids

Centroids of Lines, Areas, and Volumes

Guidelines for Choice of Elements for Integration

- Order of Element Selected for Integration
  A first order differential element should be selected in preference to a higher order element → only one integration should cover the entire figure



$A = \int dA = \int l\,dy$ $\quad A = \int\int dx\,dy$ $\quad V = \int dV = \int \pi r^2\,dy$ $\quad V = \int\int\int dxdydz$

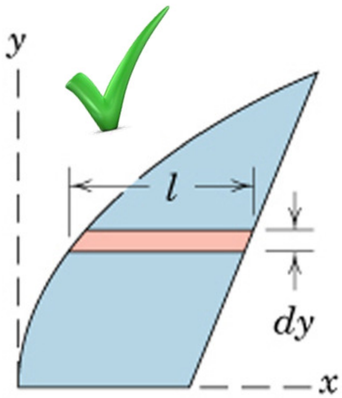
Center of Mass and Centroids

Centroids of Lines, Areas, and Volumes

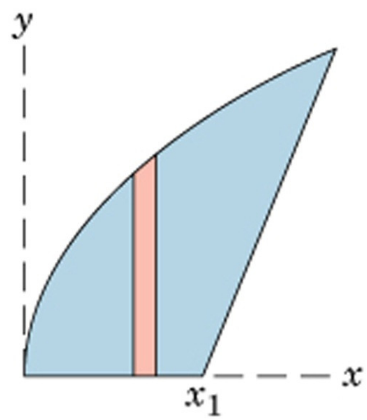
Guidelines for Choice of Elements for Integration

- Continuity

  Choose an element that can be integrated in one continuous operation to cover the entire figure → the function representing the body should be continuous
  → only one integral will cover the entire figure



Continuity in the expression for the width of the strip



Discontinuity in the expression for the height of the strip at $x = x_1$

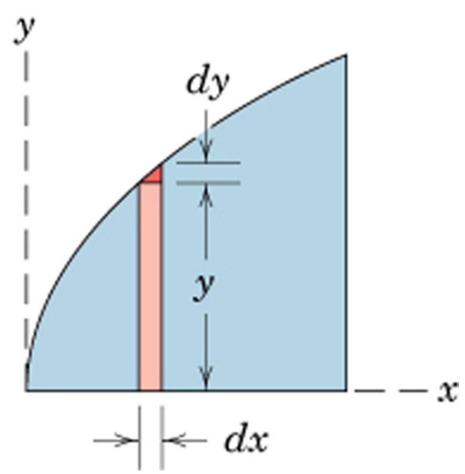
Center of Mass and Centroids

Centroids of Lines, Areas, and Volumes

Guidelines for Choice of Elements for Integration

- Discarding Higher Order Terms
  Higher order terms may always be dropped compared with lower order terms

Vertical strip of area under the curve is given by the first order term → $dA = ydx$
The second order triangular area $0.5dxdy$ may be discarded



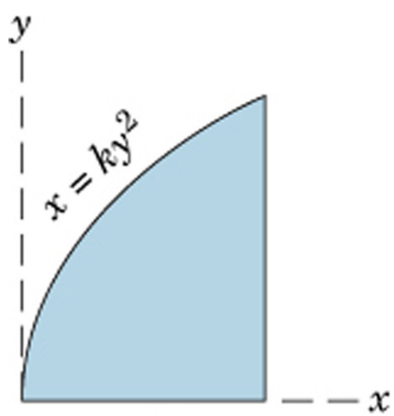
Center of Mass and Centroids

Centroids of Lines, Areas, and Volumes

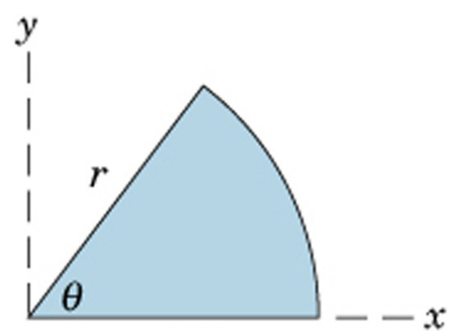
Guidelines for Choice of Elements for Integration

- Choice of Coordinates

  Coordinate system should best match the boundaries of the figure
  → easiest coordinate system that satisfies boundary conditions should be chosen



Boundaries of this area (not circular) can be easily described in rectangular coordinates



Boundaries of this circular sector are best suited to polar coordinates

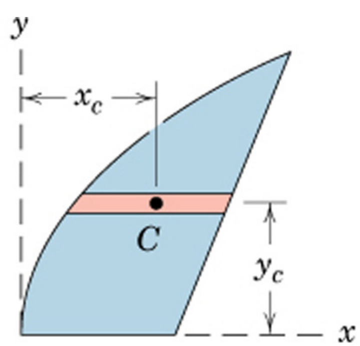
Center of Mass and Centroids

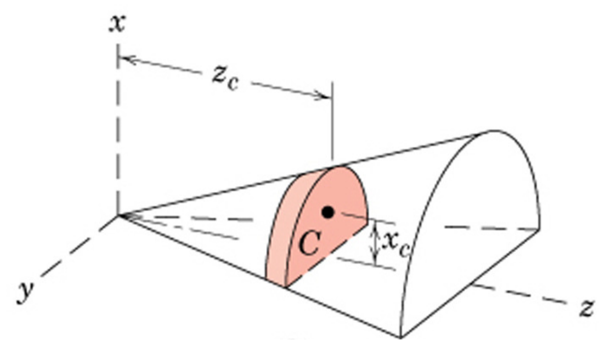
Centroids of Lines, Areas, and Volumes

Guidelines for Choice of Elements for Integration

- Centroidal Coordinate of Differential Elements

  While expressing moment of differential elements, take coordinates of the centroid of the differential element as lever arm
  (not the coordinate describing the boundary of the area)





Modified Equations

$$\bar{x} = \frac{\int x_c dA}{A} \quad \bar{y} = \frac{\int y_c dA}{A} \quad \bar{z} = \frac{\int z_c dA}{A}$$

$$\bar{x} = \frac{\int x_c dV}{V} \quad \bar{y} = \frac{\int y_c dV}{V} \quad \bar{z} = \frac{\int z_c dV}{V}$$

Center of Mass and Centroids

Centroids of Lines, Areas, and Volumes

Guidelines for Choice of Elements for Integration

1. Order of Element Selected for Integration
2. Continuity
3. Discarding Higher Order Terms
4. Choice of Coordinates
5. Centroidal Coordinate of Differential Elements

$$\bar{x} = \frac{\int x dL}{L} \quad \bar{y} = \frac{\int y dL}{L} \quad \bar{z} = \frac{\int z dL}{L}$$

$$\bar{x} = \frac{\int x_c dA}{A} \quad \bar{y} = \frac{\int y_c dA}{A} \quad \bar{z} = \frac{\int z_c dA}{A}$$

$$\bar{x} = \frac{\int x_c dV}{V} \quad \bar{y} = \frac{\int y_c dV}{V} \quad \bar{z} = \frac{\int z_c dV}{V}$$