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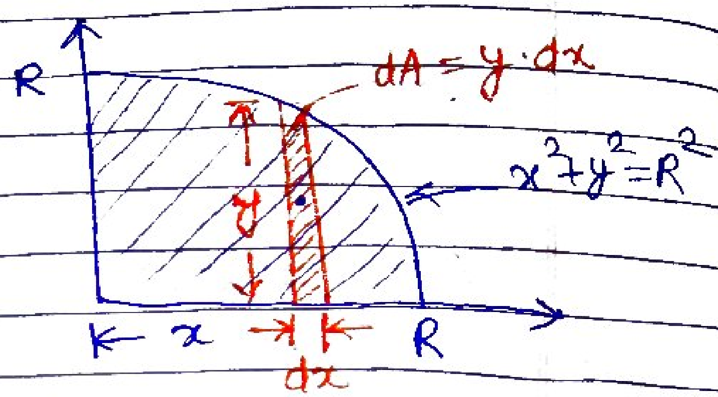
Que: Find the centroid of shaded area.

$$\bar{x} = \int \bar{x} \frac{dA}{A}$$

$$\bar{y} = \int \bar{y} \frac{dA}{A}$$



$\bar{y} = \int \bar{y} \cdot d$

$$\bar{y} = \frac{y}{2}$$

$$dA = y \cdot dx$$

$$\bar{y} = \int \bar{y} \cdot \frac{dA}{A} = \int \frac{y}{2} \cdot \frac{y\,dx}{A} = \int \frac{y^2\,dx}{2A}$$

$$x^2 + y^2 = R^2$$

$$y^2 = (R^2 - x^2)$$

$$= \frac{1}{2A} \int (R^2 - x^2)\,dx$$

$$= \frac{1}{2A} \left[\int R^2\,dx - \int x^2\,dx \right]$$

$$= \frac{1}{2A} \left[R^2 \cdot x - \frac{x^3}{3} \right]_0^R$$

$$= \frac{1}{2 \frac{1}{4} \pi R^2} \left[R^3 - \frac{R^3}{3} \right]$$

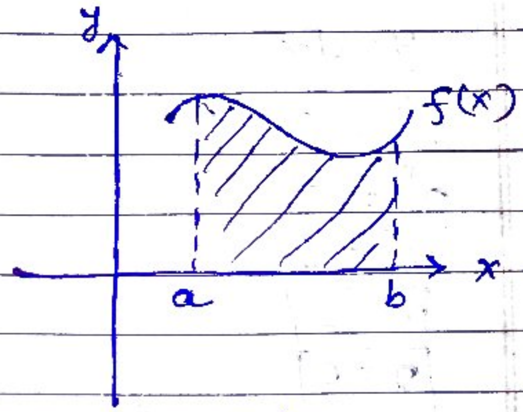
$$= \frac{2}{\pi R^2} \cdot \left[\frac{2R^3}{3} \right] = \boxed{\frac{4R}{3\pi}}$$

Same for $\bar{x}$

$$\boxed{\bar{x} = \bar{y} = \frac{4R}{3\pi}}$$

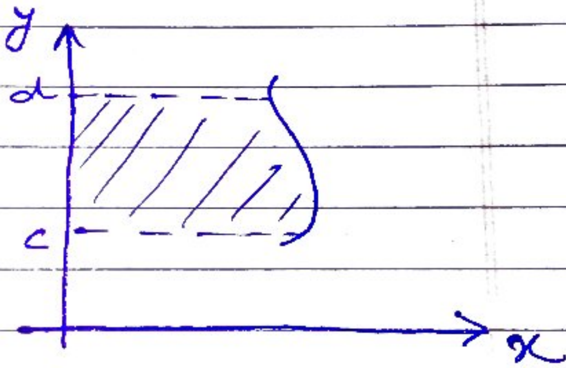
Area under any curve at x-axis :-

$$\boxed{\text{Area} = \int_{x=a}^{b} f(x)\,dx}$$



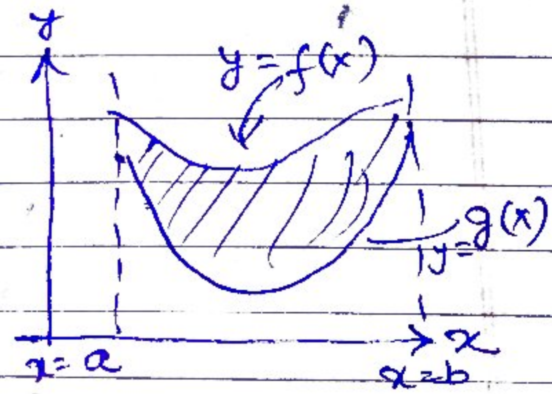
Area under any curve at y-axis

$$\boxed{\text{Area} = \int_{y=c}^{d} f(y)\,dy}$$



Area between the two curves :

$$\boxed{\text{Area} = \int_{a}^{b} [f(x) - g(x)]\,dx}$$

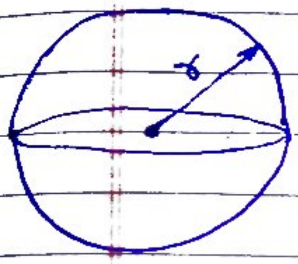


Volume of Shapes.

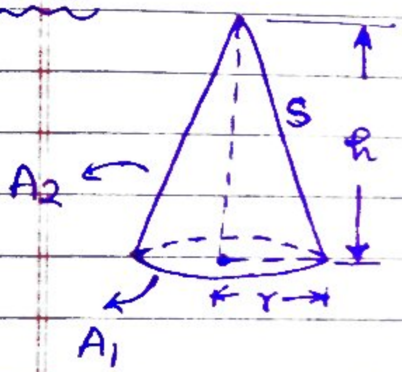
① **Sphere**

Surface Area $= 4\pi r^2$

volume $= \frac{4}{3}\pi r^3$



② **Cone**



$S \cdot A = \pi r s + \pi r^2$

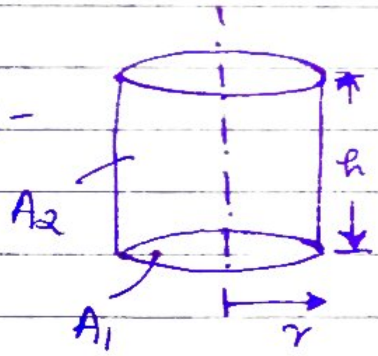
$V = \frac{1}{3}\pi r^2 h$

$S = \sqrt{r^2 + h^2}$

Area $(A_1) = \pi r^2$

$A_2 = \pi r s$

③ **Cylinder** :-



$A_1 = \pi r^2$

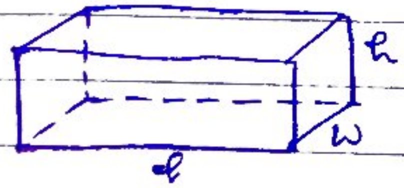
$A_2 = 2\pi r h$

$TSA = 2A_1 + A_2$

$= 2\pi r^2 + 2\pi r h$

Volume $= \pi r^2 h$

④ **Rectangular Prism** :-



$SA = 2(lw + lh + wh)$

$V = lwh$

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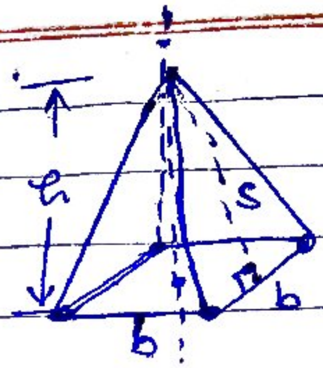
(5) Pyramid

$$A = 2bs + b^2$$

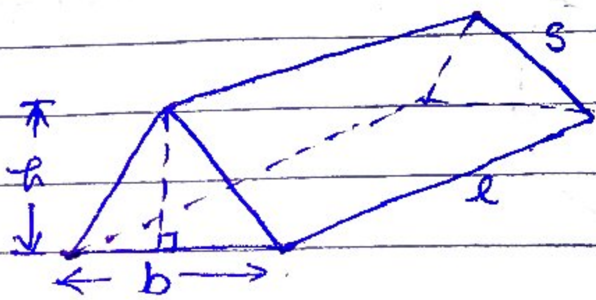
$$V = \frac{1}{3}b^2h$$



(6) Prism:-

$$SA = bh + 2ls + lb$$

$$V = \frac{1}{2}bhl$$



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Moment of Inertia of Plane area :-

- MOI is the name given to rotational inertia, the rotational analog of mass of linear motion.

- It appears in the relationships for the dynamics of rotational motion.

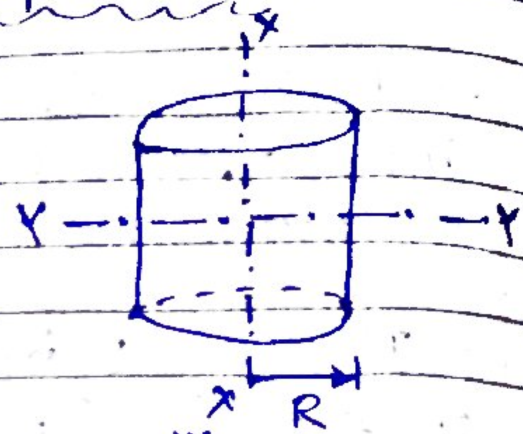
- MOI must be specified with respect to the chosen axis of rotation.

- Rotational - Linear Relationships.

	Linear motion	Rotational Motion
Position	x	θ
velocity	v	ω
Acceleration	a	α
Motion eqn	$x = vt$	$\theta = \omega t$
	$v = u + at$	$\omega = \omega_0 + \alpha t$
	$x = ut + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
	$v^2 = u^2 + 2as$	$\omega^2 = \omega_0^2 + 2\alpha\theta$
mass	m	I
Newton's IInd law	$F = ma$	$\tau = I \cdot \alpha$
momentum	$p = mv$	angular momentum $L = I\omega$
work	$W = F \cdot d$	$W = \tau \cdot \theta$
Kinetic energy	$KE = \frac{1}{2}mv^2$	$KE = \frac{1}{2}I\omega^2$
Power	$P = Fv$	$P = \tau\omega$

Some common Moments of Inertia

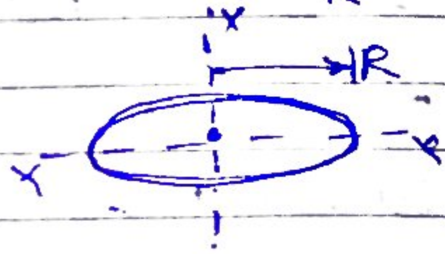
① Solid cylinder



$$I_{xx} = \frac{1}{2} MR^2$$

$$I_{YY} = \frac{1}{4} MR^2 + \frac{1}{12} ML^2$$

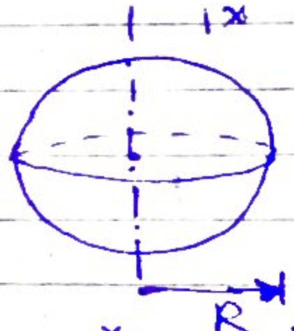
② Ring



$$I_{XX} = MR^2$$

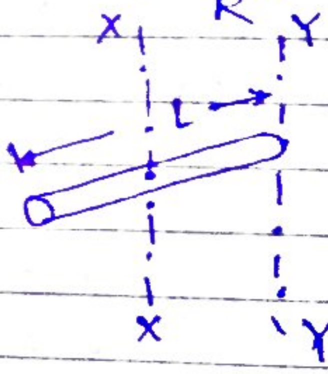
$$I_{XY} = \frac{1}{2} MR^2$$

③ Solid sphere



$$I = \frac{2}{5} MR^2$$

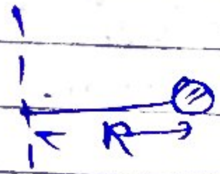
④ Rod about center



$$I_{XX} = \frac{1}{12} ML^2$$

$$I_{YY} = \frac{1}{3} ML^2$$

⑤ Point mass

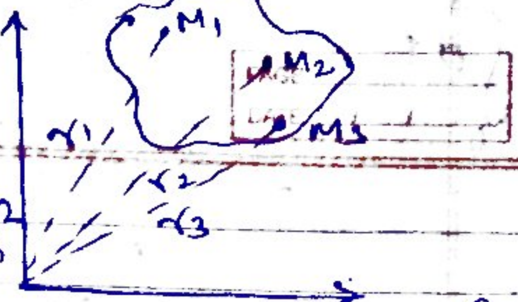


$$I = MR^2$$

General form:-

$$I = \sum_{i=1}^{n} m_n r_n^2$$

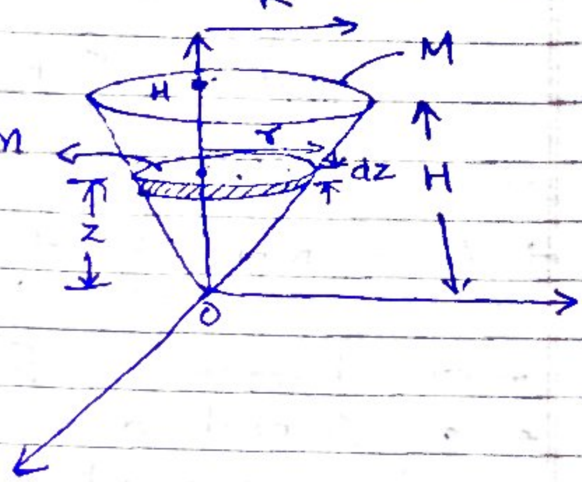
$$= m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2$$



$\rho = \frac{m}{V}$

$m = \rho \cdot V$

Que: Mass MOI of cone?



let mass = M

density = ρ

$$dm = \rho \pi r^2 \cdot dz \quad \text{—①}$$

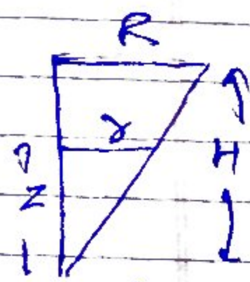
$$\rho = \frac{M}{V}$$

$$= \frac{M}{\frac{1}{3}\pi R^2 H}$$

$$\rho = \frac{3M}{\pi R^2 H} \quad \text{—②}$$

$$dm = \frac{3M}{\pi R^2 H} \cdot \pi r^2 dz = \frac{3M}{R^2 H} r^2 dz$$

$$dm = \frac{3M}{H} \frac{r^2}{R^2} dz$$



$$\frac{R}{r} = \frac{H}{z}$$

$$r = \frac{Rz}{H}$$

$$dI = \frac{1}{2} dm\, r^2$$

$$= \frac{1}{2} \frac{3M}{H} \frac{r^2}{R^2} dz\, r^2$$

$$= \frac{1}{2} 3 \frac{M}{R^2 H} \cdot r^4 dz$$

$$= \frac{1}{2} 3 \frac{M}{R^2 H} \frac{R^4 z^4}{H^4} dz = \frac{3}{2} \frac{M R^2}{\cancel{R^2} H^5} z^4 dz$$

$$\int_0^I dI = \frac{3}{2}\frac{M}{\cancel{R^2}H^5}R^2\int_0^H z^4\,dz$$

$$= \frac{3}{2}\frac{MR^2}{\cancel{R^2}H^5}\left|\frac{z^5}{5}\right|_0^H$$

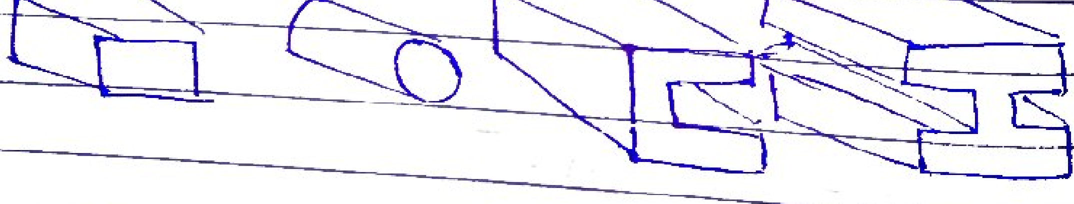
$$= \frac{3}{2}\frac{MR^2}{\cancel{R^2}\cancel{H^5}}\frac{\cancel{H^5}}{5} = \frac{3}{10}\frac{MR^2}{\cancel{R^2}}$$

$$\boxed{I = \frac{3}{10}MR^2}$$

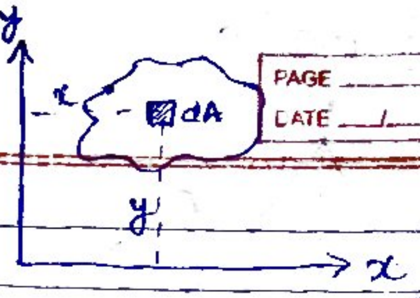
Area Moment of Inertia :-

- also Known as Second moment of area: or Second area moment.
- is a geometrical property of an area.
- it reflects how its points are distributed with regards to an arbitrary axis.
- Denoted by I or J.
- ~~It is for axis perpendicular to the plane.~~

- It is distribution of area. wrt to axis.
- Used in Beams.
- .



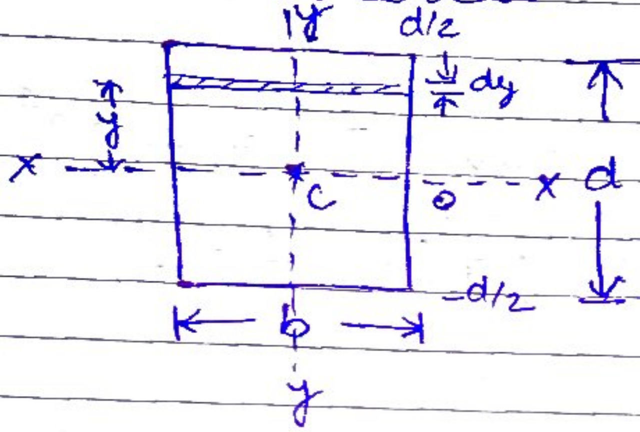
Expression :-



$$I_{xx} = \int y^2 dA = \iint y^2 \, dx \, dy$$

$$I_{yy} = \int x^2 dA = \iint x^2 \, dx \, dy$$

(1) Rectangular Section :-



I_{xx} = centroidal axis.

$$I_{xx} = \int y^2 \, dA$$

$$= \int y^2 \, b\,dy$$

$$= 2\int_0^{d/2} y^2 \, b\,dy$$

$$= 2b \left[\frac{y^3}{3} \right]_0^{d/2}$$

$$= \frac{\cancel{2}}{3} b \frac{d^3}{\cancel{8}\,4}$$

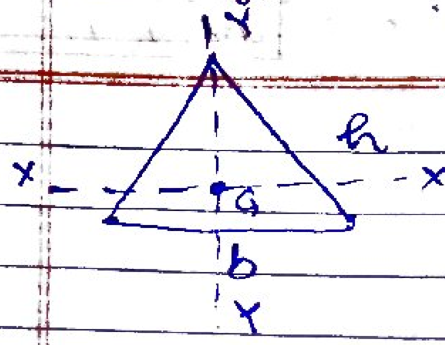
$$\boxed{I_{xx} = \frac{bd^3}{12}}$$

Similarly, $\boxed{I_{yy} = \frac{db^3}{12}}$ (mm^4) unit

$$\text{Polar MOI} = (I_C) = \frac{1}{12}(b^2 + h^2)\,bh$$

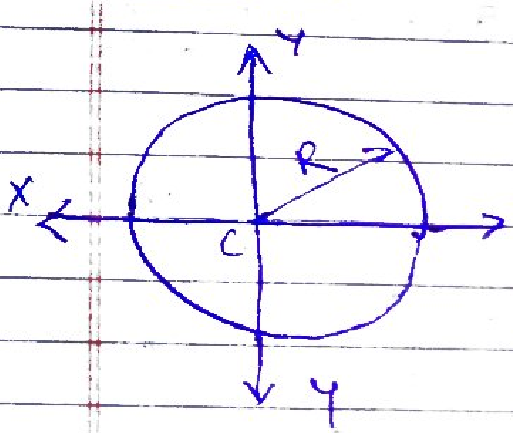
$$\boxed{I_C = I_{xx} + I_{yy}}$$

② Triangle



$$I_{xx} = \frac{bh^3}{36}$$

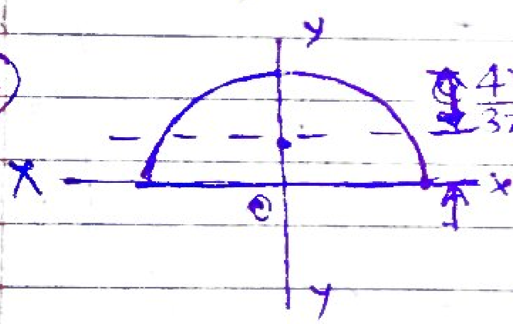
③ Circle :-



$$I_{xx} = I_{yy} = \frac{1}{4}\pi R^4$$

$$\text{Polar } (I_c) = \frac{1}{2}\pi R^4$$

④

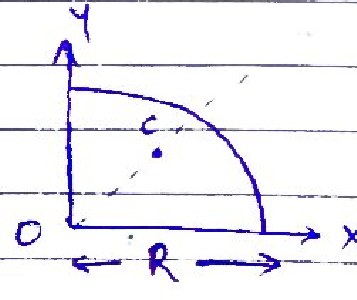


$\frac{4r}{3\pi}$

$$I_{yy} = I_{xx} = \frac{1}{8}\pi R^4$$

$$\text{Polar}(I_0) = \frac{1}{4}\pi R^4$$

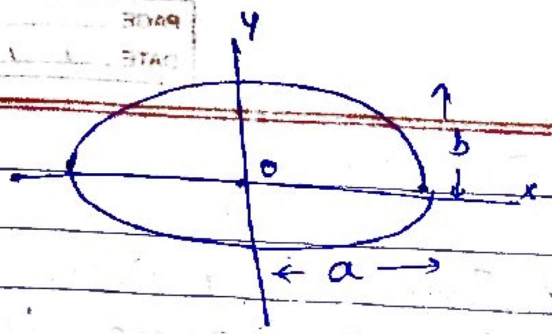
⑤



$$I_x = I_y = \frac{1}{16}\pi R^4$$

$$\text{Polar} = (I_0) = \frac{1}{8}\pi R^4$$

(6)



$$I_x = \frac{1}{4}\pi a b^3$$

$$I_y = \frac{1}{4}\pi a^3 b$$

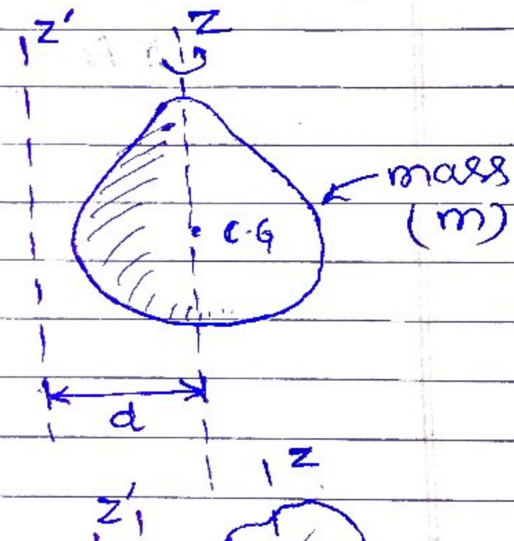
$$J_0 = \frac{1}{4}\pi ab(a^2+b^2)$$

Parallel axis Theorem :-

- also known as Huygen's-Steiner Theorem
- Can be used to determine the mass MOI or Second MOA of any rigid body
-

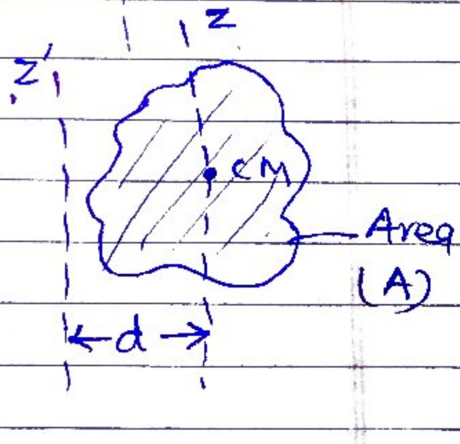
For Mass moment of Inertia :-

$$\boxed{I_{z'} = I_z + md^2}$$



For Second MOA

$$\boxed{I_{z'} = I_z + Ad^2}$$

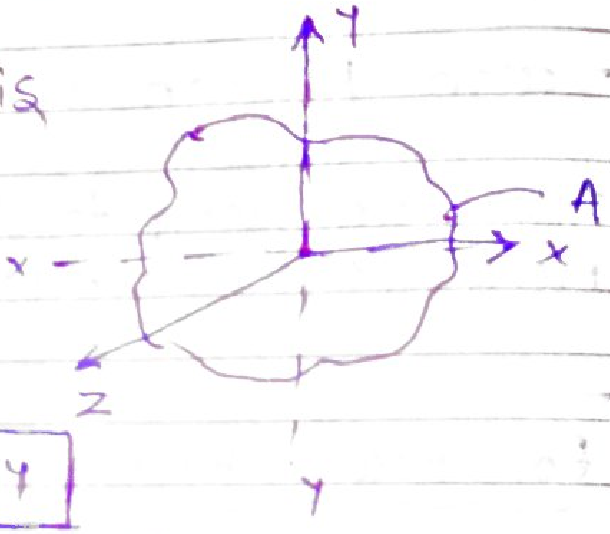


Perpendicular axis Theorem :-

It state that, "If I_{xx} & I_{yy} are the MOI of a given plane figure then there ~~these~~ is also MOI which is ⊥ to the plane figure and both I_{xx} & I_{yy}.

→ also called Polar axis theorem. and

→ and MOI about z axis



$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$

When I_{zz} → Polar MOI.