

# Remedial class on solving Complex Quantities (Transformer circuit).

Transformer coil impedance  $R_{eq} + jX_{eq}$   
Transformer magnetic branch (shunt branch)

$$\text{impedance } Z_m = \frac{1}{Y_m} \leftarrow \text{admittance}$$

$$\text{where } Y_m = G_c + S_m$$

$$\rightarrow \text{conductance (core)} = \frac{1}{r_c}$$

$$\rightarrow \text{susceptance} = \frac{1}{jX_m}$$

Now these quantities are similar to complex quantities and all the applicable laws of complex algebra are applicable here.

For your ready reference

A complex quantity  $a + jb$  can be expressed in several ways:—

(1)  $(a + jb)$  Rectangular form,  $a$  is real,  $b$  is imaginary

or  
(2)  $|a + jb| \angle \theta$  where  $|a + jb| = \sqrt{a^2 + b^2}$  and  $\theta = \tan^{-1} \frac{b}{a}$   
this is angular form

(3)  $A(\cos \theta + j \sin \theta) \rightarrow$  Trigonometrical form  
or  $A = \sqrt{a^2 + b^2}$  and  $\theta = \tan^{-1} \frac{b}{a}$

(4)  $A e^{j\theta} \rightarrow$  Exponential form  
where  $A = \sqrt{a^2 + b^2}$  .  $\theta = \tan^{-1} \frac{b}{a}$

② It is convenient to divide and multiply quantities in Angular form as

$$A \angle \theta_1 \times B \angle \theta_2 = AB \angle \theta_1 + \theta_2$$

$$\text{and } \frac{A \angle \theta_1}{B \angle \theta_2} = \frac{A}{B} \angle \theta_1 - \theta_2$$

It is convenient to add and subtract quantities in rectangular form as

$$(A_1 + jB_1) + (A_2 + jB_2) \\ = (A_1 + A_2) + j(B_1 + B_2)$$

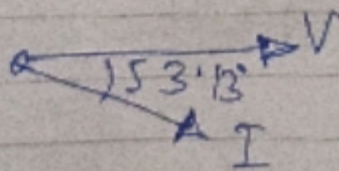
$$\text{and } (A_1 + jB_1) - (A_2 + jB_2) \\ = (A_1 - A_2) - j(B_1 - B_2)$$

If voltage is given 415 volts means it is 415 volts RMS and to be taken as reference (so its angle is zero)

$$V = 415 \angle 0^\circ = 415 (\cos 0^\circ + j \sin 0^\circ) \\ = 415 (1 + j0) \text{ V}$$

$$Z = 3 + j4 \ \Omega = \sqrt{3^2 + 4^2} \angle \tan^{-1} \frac{4}{3} = 5 \angle 53.13^\circ$$

$$\text{Find current } I, \quad I = \frac{V}{Z} = \frac{415 \angle 0^\circ}{5 \angle 53.13^\circ}$$



$$= 83 \angle 0 - 53.13^\circ \text{ A (Angular)} \\ = 83 (0.6 - 0.8j) \\ = 49.8 - j66.4 \text{ A (Rectangular)}$$

Question: The input impedance and current of a two terminal circuit are  $\bar{V} = 10 \angle 23.6^\circ \text{ V}$  and  $\bar{I} = 0.25 \angle 65^\circ \text{ A}$

Find the circuit impedance and its components.

Solution:

The impedance in polar (angular) form

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{10 \angle 23.6^\circ}{0.25 \angle 65^\circ} = 40 \angle -41.4^\circ \Omega$$

The above can be written as

$$\bar{Z} = \text{Re}(40 \angle -41.4^\circ) + j \text{Im}(40 \angle -41.4^\circ)$$

$$\text{or } \bar{Z} = 40 [\cos(-41.4^\circ) + j \sin(-41.4^\circ)]$$

$$= 40 \underline{\underline{F}}$$

$$= 30 + j(-26.5) \Omega$$

So  $R = \text{Real part} = 30 \Omega$  (Resistance)

$X = \text{Imaginary part} = -26.5 \Omega$

(Inductive Capacitive Reactance)

The above example is from single phase. It is taken here to illustrate the operation of complex numbers which is asked by many of you. Hope this topic is clear!!