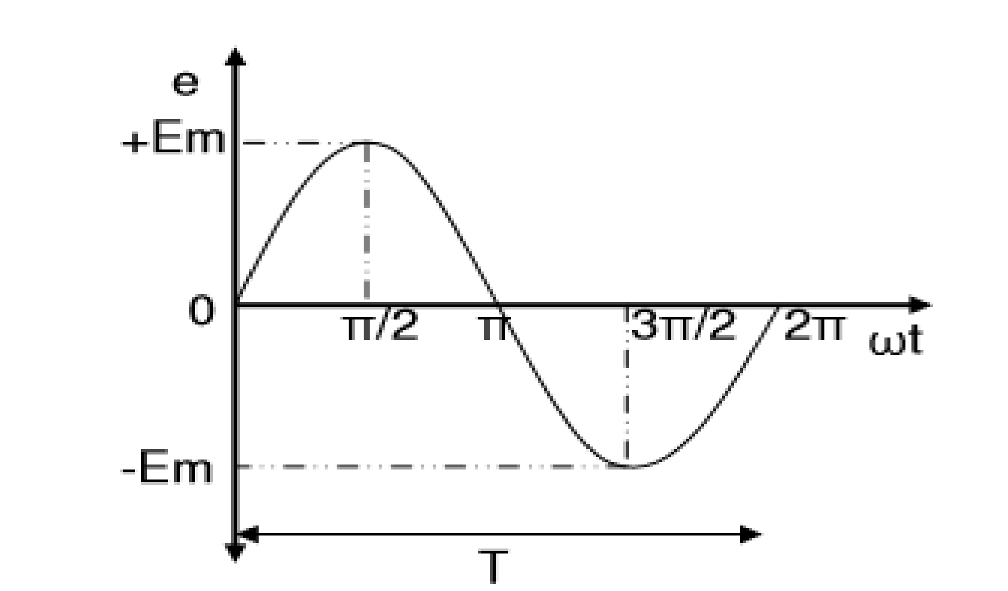
Basic Electrical Engineering
KEE101

Department of Engineering
Uttar Pradesh Textile Technology Institute
Session 2019-20
Semester-II
Faculty: Dr Indra Prakash Mishra

Single Phase AC



Definition of Alternating Quantity

An alternating quantity changes continuously in magnitude and alternates in direction at regular intervals of time.

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Single Phase AC: Important Terms

Important terms associated with an alternating quantity are defined below.

1. Amplitude

It is the maximum value attained by an alternating quantity. Also called as maximum or peak value

2. Time Period (T)

It is the Time Taken in seconds to complete one cycle of an alternating quantity

3. Instantaneous Value

It is the value of the quantity at any instant

4. Frequency (f)

It is the number of cycles that occur in one second. The unit for frequency is Hz or cycles/sec.

The relationship between frequency and time period can be derived as follows.

Time taken to complete f cycles = 1 second

Time taken to complete 1 cycle = 1/f second

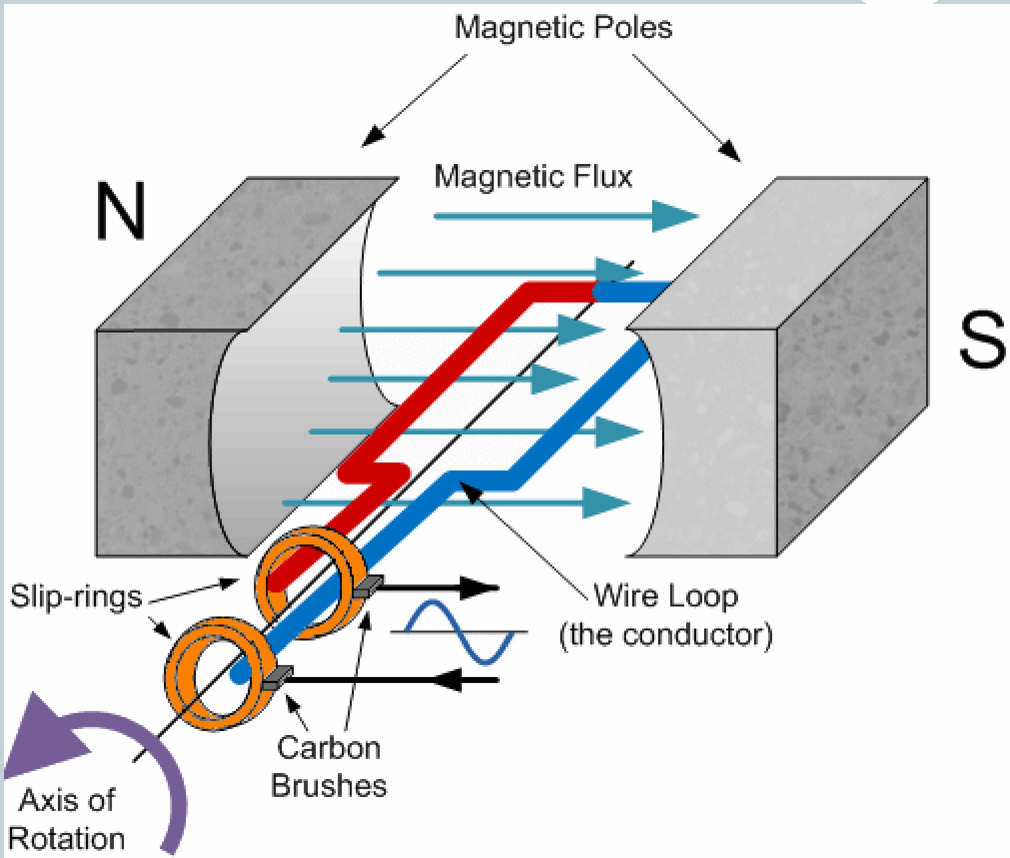
$$T = 1/f$$

Single Phase AC

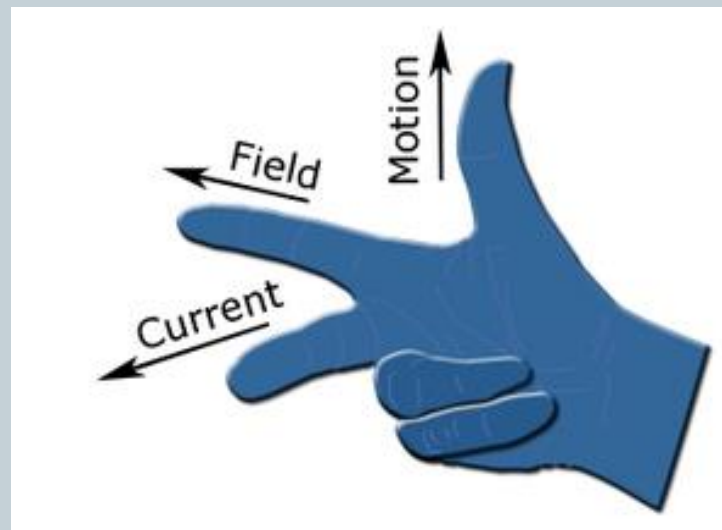
<u>Advantages of AC system over DC system</u>

1. AC voltages can be efficiently stepped up/down using transformer
2. AC motors are cheaper and simpler in construction than DC motors
3. Switchgear for AC system is simpler than DC system

Single Phase AC: Generation

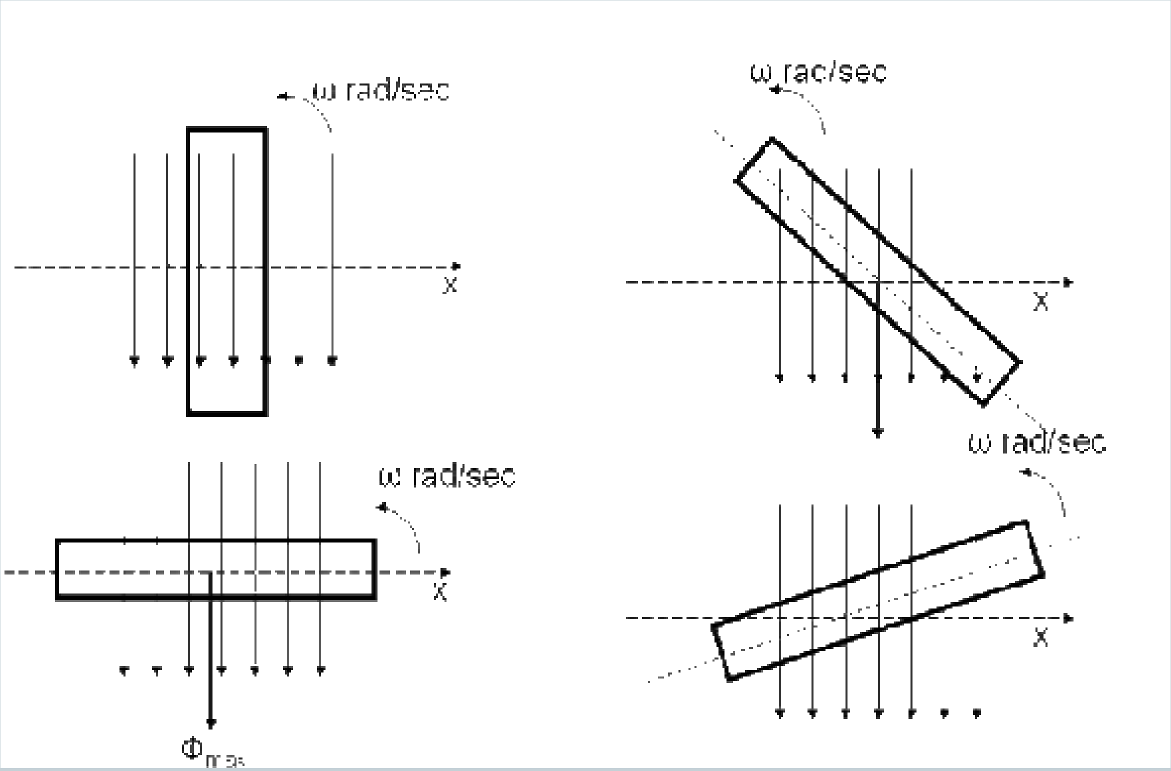


- It can be generated by Linking a coil with varying flux.



Single Phase AC: Generation

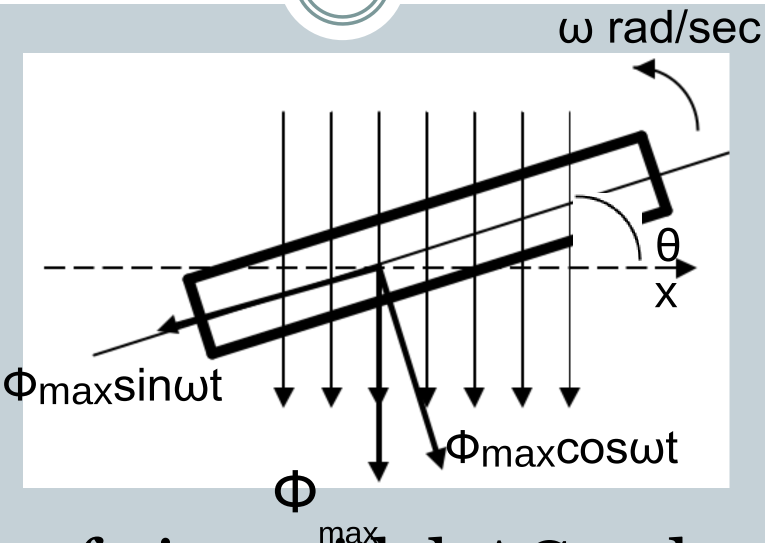
Consider a rectangular coil of N turns placed in a Uniform Magnetic figure as shown in figure:



Single Phase AC: Generation

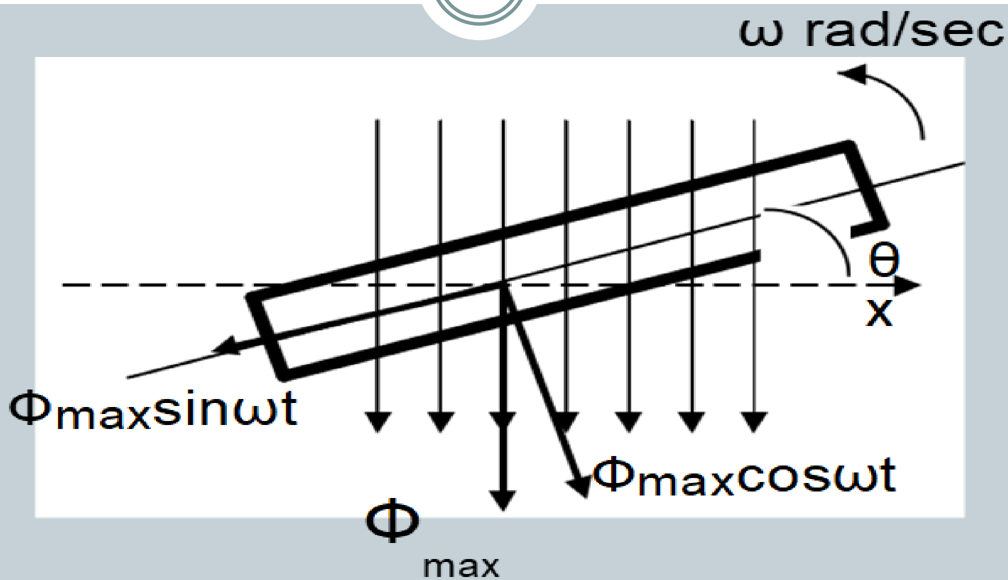
When the coil is in the vertical position, the flux linking the coil is zero because the plane of the coil is parallel to the direction of the magnetic field. Hence at this position, the emf induced in the coil is zero. When the coil moves by some angle in the anticlockwise direction, there is a rate of change of flux linking the coil and hence an emf is induced in the coil. When the coil reaches the horizontal position, the flux linking the coil is maximum, and hence the emf induced is also maximum. When the coil further moves in the anticlockwise direction, the emf induced in the coil reduces. Next when the coil comes to the vertical position, the emf induced becomes zero. After that the same cycle repeats and the emf is induced in the opposite direction. When the coil completes one complete revolution, one cycle of AC voltage is generated.

Single Phase AC: Generation



- The generation of sinusoidal AC voltage can also be explained using mathematical equations.
- Consider a rectangular coil of N turns placed in a uniform magnetic field in the position shown in the figure.

Single Phase AC: Generation



- The maximum flux linking the coil is in the downward direction as shown in the figure.
- This flux can be divided into two components, one component acting along the plane of the coil $\Phi_{max}\sin\omega t$ and another component acting perpendicular to the plane of the coil $\Phi_{max}\cos\omega t$.

Single Phase AC: Generation

The component of flux acting along the plane of the coil does not induce any flux in the coil. Only the component acting perpendicular to the plane of the coil ie $\Phi_{max}\cos\omega t$ induces an emf in the coil.

$$\Phi = \Phi_{max} \cos \omega t$$

$$e = -N \frac{d\Phi}{dt}$$

$$e = -N \frac{d}{dt} \Phi_{max} \cos \omega t$$

$$e = N\Phi_{max} \omega \sin \omega t$$

$$e = E_m \sin \omega t$$

Hence the emf induced in the coil is a sinusoidal emf. This will induce a sinusoidal current in the circuit given by

$$i = I_m \sin \omega t$$

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Single Phase AC: Solved Problem

Solved Problem 1

An alternating current i is given by i = 141.4 sin 314t Find

i) The maximum value

ii) Frequency

iii) Time Period

iv) The instantaneous value when t=3ms, i = 141.4 sin 314t

Ans: $i = I_m \sin \omega t$

Maximum value Im=141.4 V

ω = 314 rad/sec

T=1/f = 0.02 sec

i=141.4 sin(314x0.003) = 114.35A

Single Phase AC: Average Value

The arithmetic average of all the values of an alternating quantity over one cycle is called its average value

$$V_{av} = \frac{Area\ Under\ the\ curve\ over\ half\ cycle}{Base}$$

$$V_{av} = \frac{\int_0^{\pi} v_i\, d\omega t}{\pi}$$

$$Average\ Value\ V_{av} = \frac{2V_m}{\pi}$$ for sinusoidal wave

$$Average\ Value\ V_{av} = 0.637V_m$$

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Single Phase AC: Average Value

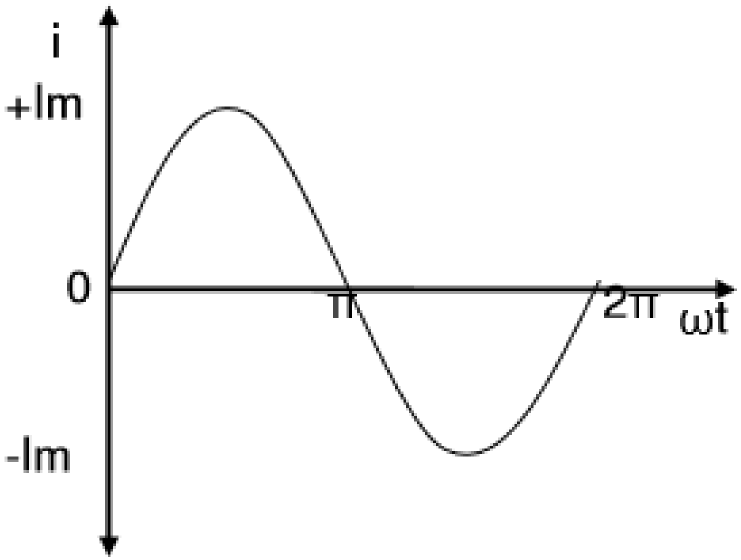
For Symmetrical waveforms, the average value calculated over one cycle becomes equal to zero because the positive area cancels the negative area. Hence for symmetrical waveforms, the average value is calculated for half cycle.

$$\text{Average value} = \frac{\text{Area under one half cycle}}{\text{Base}}$$

$$V_{av} = \frac{1}{\pi}\int_0^{\pi} v\,d(\omega t)$$

Single Phase AC: Average Value

Average value of a sinusoidal current



$$i = I_m \sin \omega t$$

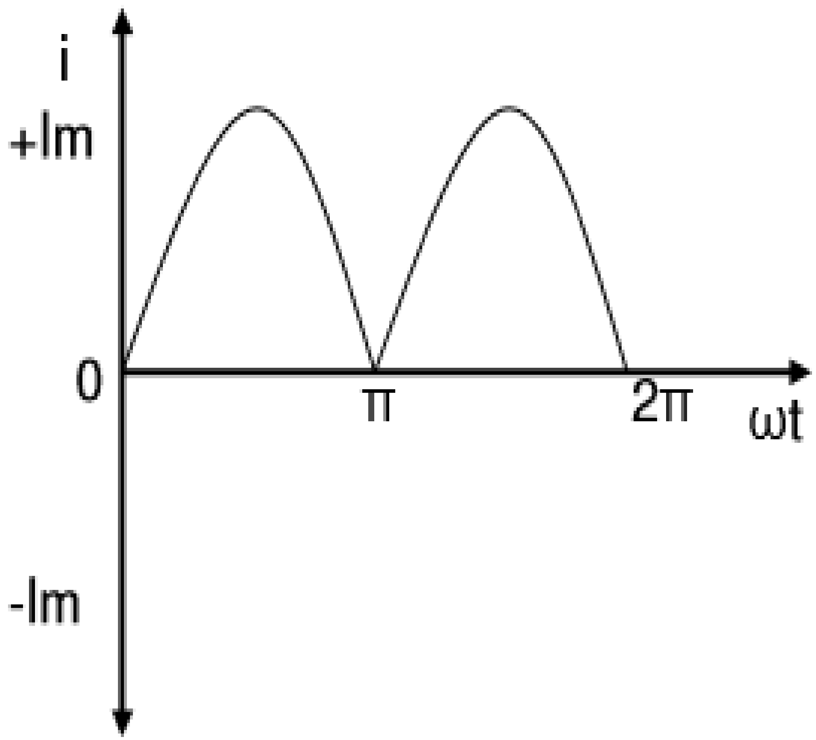
$$I_{av} = \frac{1}{\pi}\int_0^{\pi} i\,d(\omega t)$$

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} I_m \sin \omega t\,d(\omega t)$$

$$I_{av} = \frac{2I_m}{\pi} = 0.637 I_m$$

Single Phase AC: Average Value

Average value of a full wave rectifier output



$$i = I_m \sin \omega t$$

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} i\,d(\omega t)$$

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} I_m \sin \omega t\,d(\omega t)$$

$$I_{av} = \frac{2I_m}{\pi} = 0.637 I_m$$

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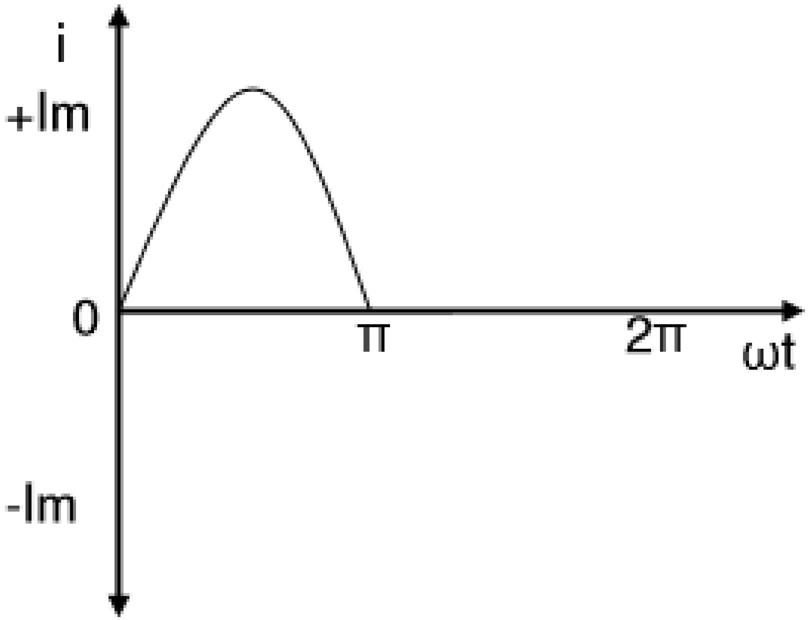
Single Phase AC: Average Value

Average value of a half wave rectifier output



$$i = I_m \sin \omega t$$

$$I_{av} = \frac{1}{2\pi}\int_0^{2\pi} i\,d(\omega t)$$

$$I_{av} = \frac{1}{2\pi}\int_0^{\pi} I_m \sin \omega t\,d(\omega t)$$

$$I_{av} = \frac{I_m}{\pi} = 0.318 I_m$$

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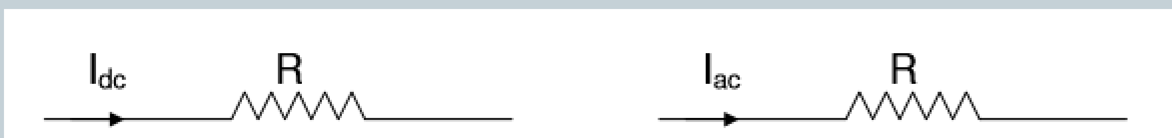
Single Phase AC: RMS Value

RMS or Effective Value

The effective or RMS value of an alternating quantity is that steady current (dc) which when flowing through a given resistance for a given time produces the same amount of heat produced by the alternating current flowing through the same resistance for the same time.

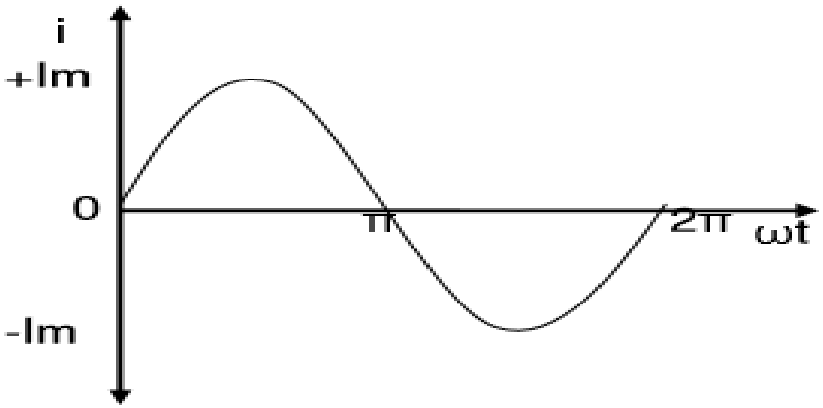


$$RMS = \sqrt{\frac{\text{Area under squared curve}}{base}}$$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_{0}^{2\pi} v^2 d(\omega t)}$$

Single Phase AC: RMS Value

RMS value of a sinusoidal current



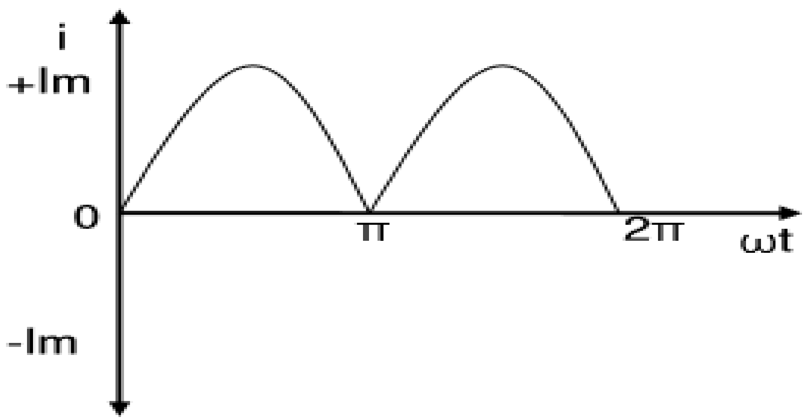
$$i = I_m \sin \omega t$$

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 d(\omega t)}$$

$$I_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2 \omega t d(\omega t)}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

RMS value of a full wave rectifier output



$$i = I_m \sin \omega t$$

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 d(\omega t)}$$

$$I_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2 \omega t d(\omega t)}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

Single Phase AC: RMS Value

$$Rms\ Value\ V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$Rms\ Value\ V_{rms} = 0.707V_m$$

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Single Phase AC: Form Factor

The Ratio of RMS Value to the average value of the alternating quantity is Form Factor.

$$FF = \frac{RMS\ Value}{Average\ Value}$$

Single Phase AC: Form Factor

$$\text{Form Factor of Sinusoid} = \frac{I_{rms}}{I_{av}}$$

$$\text{Form Factor of Sinusoid} = \frac{0.707 I_m}{0.637 I_m}$$

$$\text{Form Factor of Sinusoid} = 1.11$$

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Single Phase AC: Peak Factor

The ratio of Maximum value to the RMS value of the alternating quantity is Peak Factor.

$$PF = \frac{Maximum\ Value}{RMS\ Value}$$

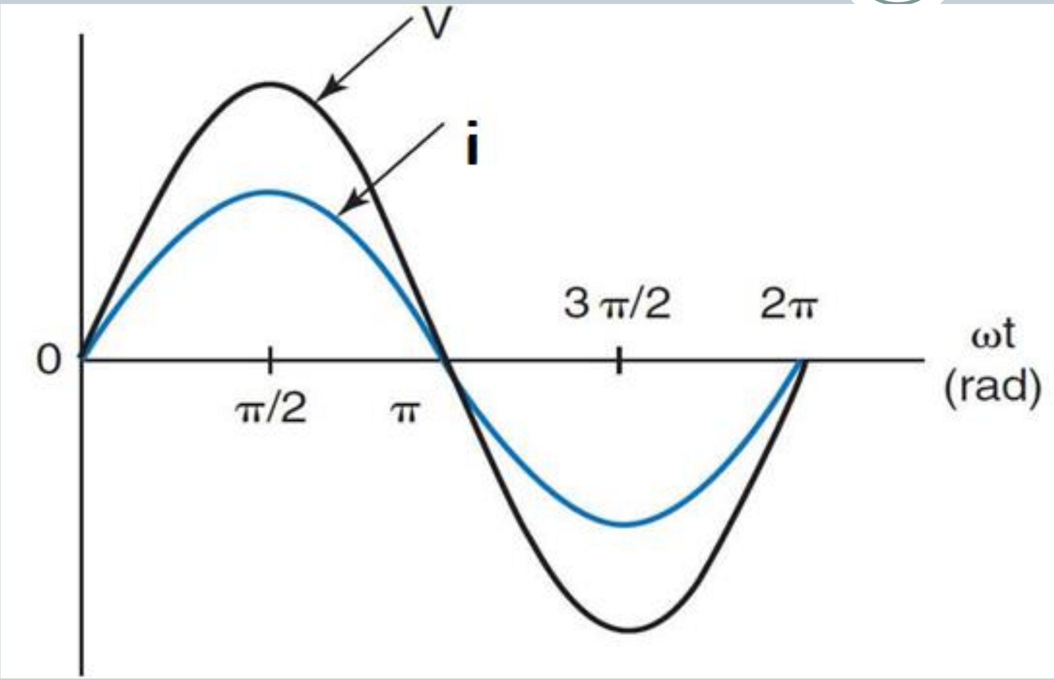
Single Phase AC: Peak Factor

$$\text{PeakFactor of Sinusoid} = \frac{I_{max}}{I_{rms}}$$

$$\text{PeakFactor of Sinusoid} = \frac{I_m}{0.707 I_m}$$

$$\text{PeakFactor of Sinusoid} = 1.414$$

Single Phase AC : In Phase



The Waveforms

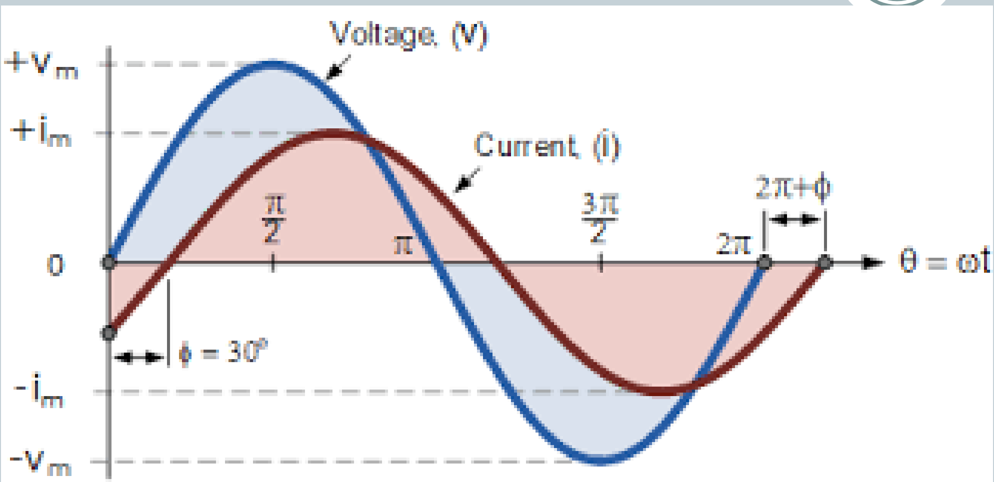
Two waveforms are said to be in phase, when the phase difference between them is zero. That is the zero points of both the waveforms are same.

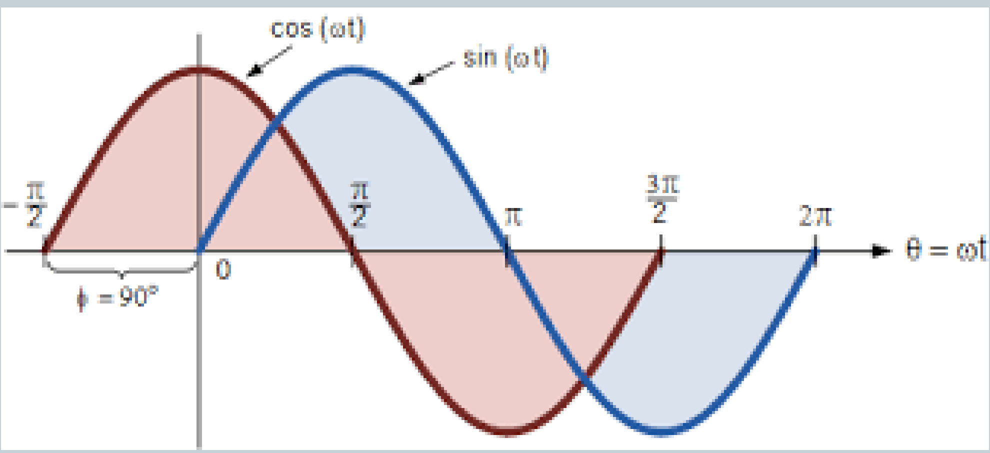
The Equations

$$\mathbf{v} = V_m \sin \omega t$$

$$\mathbf{i} = I_m \sin \omega t$$

Single Phase AC : Lagging or Leading





In the figure shown, the zero point of the current waveform is after the zero point of the voltage waveform.

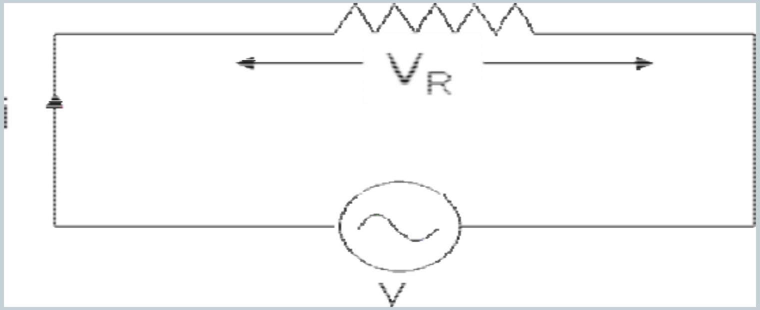
Hence

the current is lagging behind the voltage.

Alternatively

Voltage wave is leading the current wave.

AC circuit with a pure resistance



Consider an AC circuit with a pure resistance R as shown in the figure. The alternating voltage v is given by

$$v = V_m \sin \omega t \quad \text{---------- (1)}$$

The current flowing in the circuit is i. The voltage across the resistor is given as V_R which is the same as v.

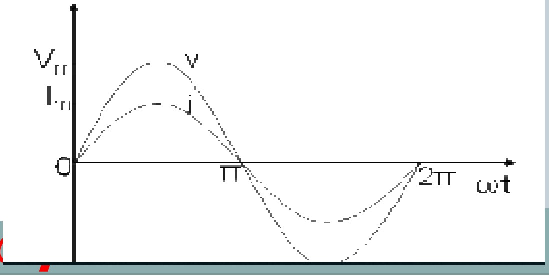
Using ohms law, we can write the following relations

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R}$$

$$i = I_m \sin \omega t \quad \text{---------------(2)}$$

Where $I_m = \frac{V_m}{R}$

From equation (1) and (2) we conclude that in a pure resistive circuit, the voltage and current are in phase. Hence the voltage and current waveforms and phasors can be drawn as below.



AC circuit with a pure resistance

Instantaneous power

The instantaneous power in the above circuit can be derived as follows

$$p = vi$$

$$p = (V_m \sin \omega t)(I_m \sin \omega t)$$

$$p = V_m I_m \sin^2 \omega t$$

$$p = \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$

$$p = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$$

The instantaneous power consists of two terms. The first term is called as the constant power term and the second term is called as the fluctuating power term.

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AC circuit with a pure resistance

Average power

From the instantaneous power we can find the average power over one cycle as follows

$$P = \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t\right] d\omega t$$

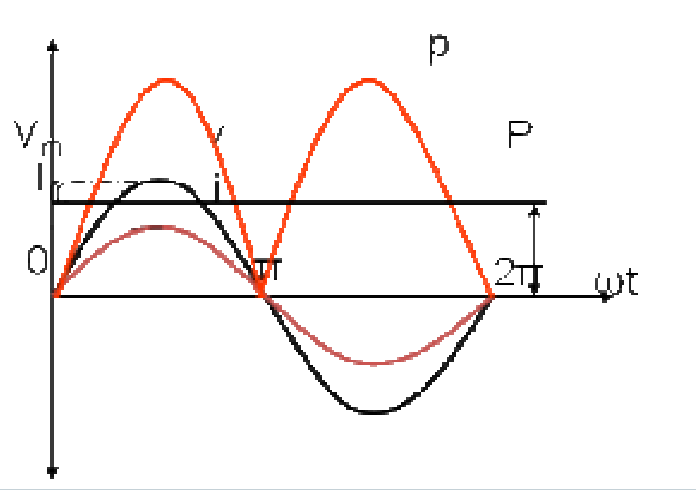
$$P = \frac{V_m I_m}{2} - \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{V_m I_m}{2}\cos 2\omega t\right] d\omega t$$

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}$$

$$P = V.I$$

As seen above the average power is the product of the rms voltage and the rms current.

The voltage, current and power waveforms of a purely resistive circuit is as shown in the figure.



AC circuit with a pure resistance

- Problem 2: An ac circuit consists of a pure resistance of 10 and is connected to an ac supply of 230 V, 50 Hz. Calculate the (i) current (ii) power consumed and (iii) equations for voltage and current.

$$(i) I = \frac{V}{R} = \frac{230}{10} = 23A$$

$$(ii) P = VI = 230 \times 23 = 5260W$$

$$(iii) V_m = \sqrt{2}V = 325.27V$$

$$I_m = \sqrt{2}I = 32.52A$$

$$\omega = 2\pi f = 314 rad / \sec$$

$$v = 325.25 \sin 314t$$

$$i = 32.52 \sin 314t$$

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Single Phase AC: Resonance

- Resonance is special circuit condition in Electrical Engineering when the response is maximum.
- For the resonance the circuit need to have inductor and capacitor. There may be circuit (Coil) resistance R also
- Series Resonance Circuit

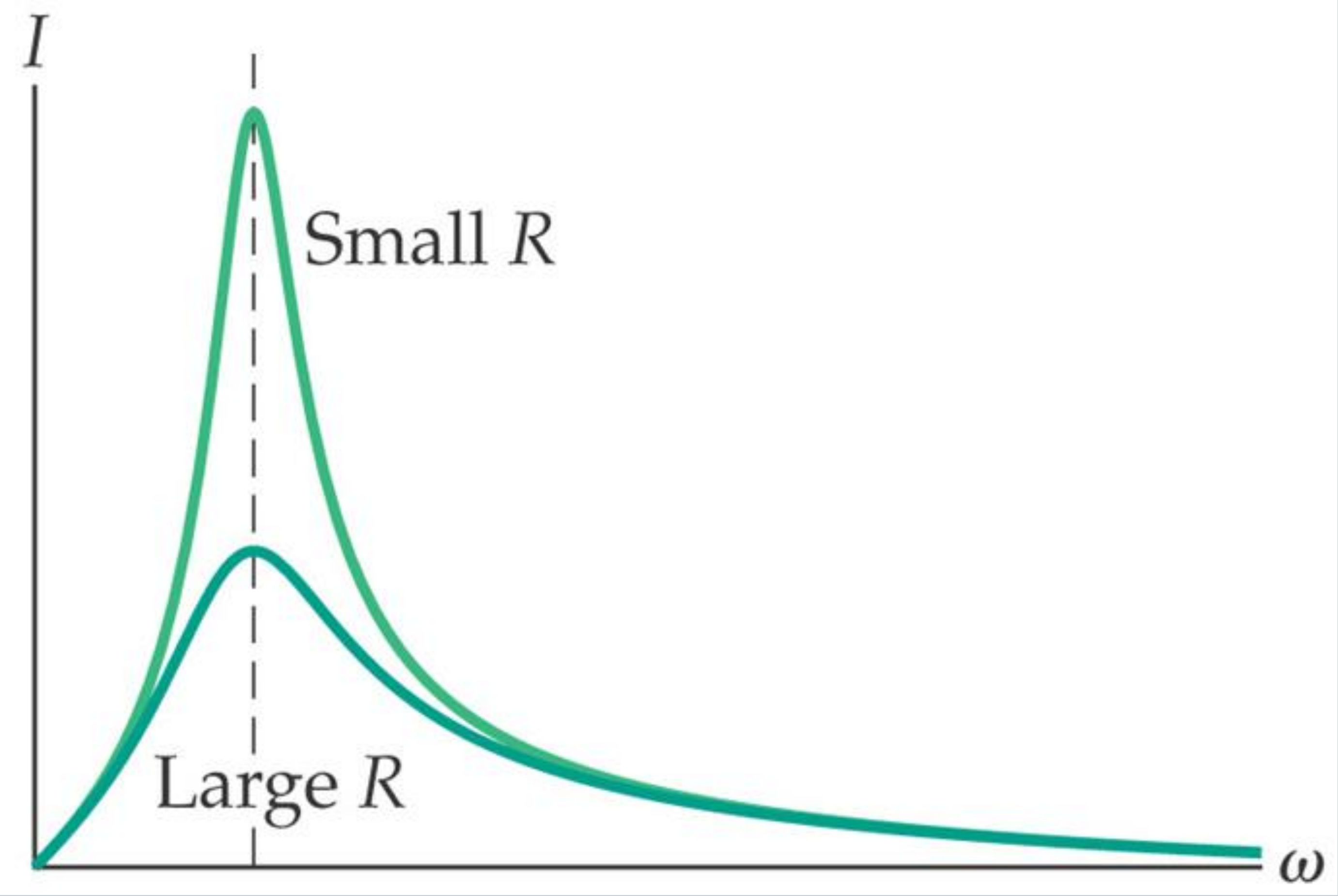
    Also known as Voltage amplifier

    Also known as Acceptor Circuit
- Parallel Resonance Circuit

    Also known as Current amplifier

    Also known as Rejecter Circuit

Single Phase AC : Resonance



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Single Phase AC: Resonance
Self Revision topics

<u>For Series RLC Circuit:(Already Explained in Class)</u>

- Impedance at resonance
- Half Power current in terms of Resonance current
- Band width
- Edge or Corner frequencies
- Derivation of expression for finding out edge frequencies.
- Quality factor of a coil.
- Graphical Representation of Resonance

Single Phase AC: Text Book and Reference Books

Text Book

(1) I. J. Nagrath and D. P. Kothari

Basic Electrical Engineering- Tata McGraw Hill

Reference Books:

(2) Alexander S. Langsdorf

Theory of Alternating Current Machinery-TMGH

(3) D. P. Kothari , I. J. Nagrath

Electric Machines-Tata McGraw Hill