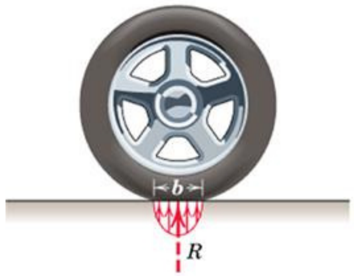
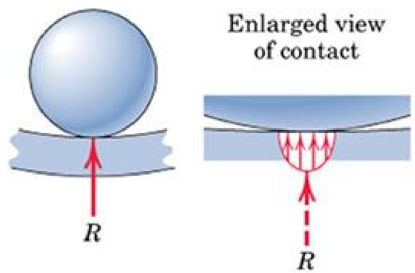
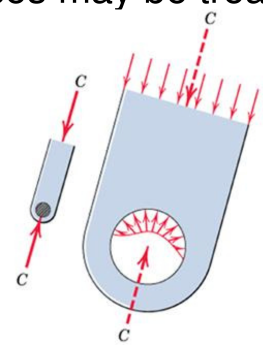
Center of Mass and Centroids

Concentrated Forces: If dimension of the contact area is negligible compared to other dimensions of the body → the contact forces may be treated as Concentrated Forces

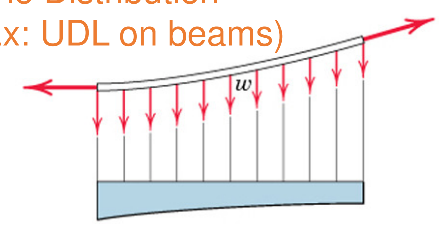


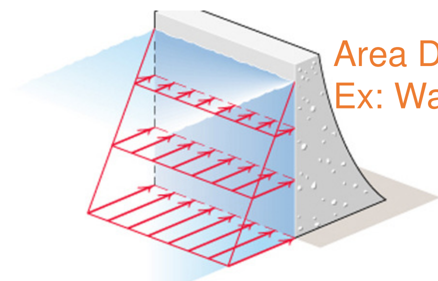




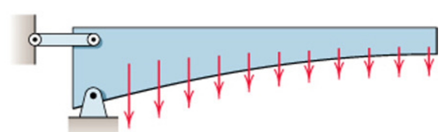
Distributed Forces: If forces are applied over a region whose dimension is not negligible compared with other pertinent dimensions → proper distribution of contact forces must be accounted for to know intensity of force at any location.

Line Distribution
(Ex: UDL on beams)





Area Distribution
Ex: Water Pressure



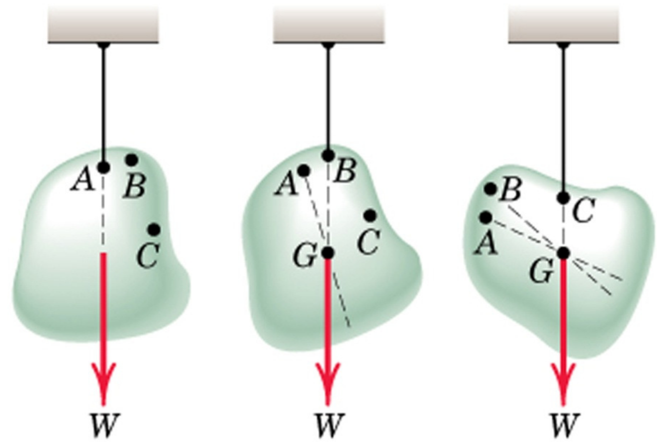
Body Distribution
(Ex: Self weight)

Center of Mass and Centroids

Center of Mass

A body of mass *m* in equilibrium under the action of tension in the cord, and resultant *W* of the gravitational forces acting on all particles of the body.

- The resultant is collinear with the cord



Suspend the body from different points on the body

- Dotted lines show lines of action of the resultant force in each case.
- These lines of action will be concurrent at a single point G

As long as dimensions of the body are smaller compared with those of the earth.
- we assume uniform and parallel force field due to the gravitational attraction of the earth.

The unique **Point G** is called the Center of Gravity of the body (CG)

Center of Mass and Centroids

Determination of CG

- Apply Principle of Moments

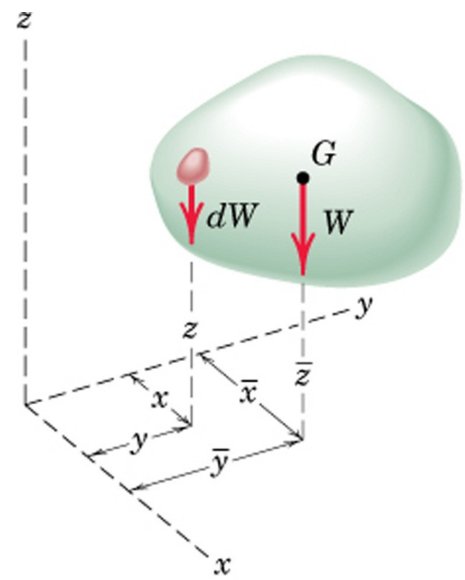
Moment of resultant gravitational force W about any axis equals sum of the moments about the same axis of the gravitational forces dW acting on all particles treated as infinitesimal elements.

Weight of the body $W = \int dW$

Moment of weight of an element (dW) @ x-axis = ydW

Sum of moments for all elements of body = $\int ydW$

From Principle of Moments: $\int ydW = \bar{y}\ W$



$$\bar{x} = \frac{\int xdW}{W} \qquad \bar{y} = \frac{\int ydW}{W} \qquad \bar{z} = \frac{\int zdW}{W}$$

→ Numerator of these expressions represents the sum of the moments; Product of W and corresponding coordinate of G represents the moment of the sum → Moment Principle.

Center of Mass and Centroids

$$\bar{x} = \frac{\int x dW}{W} \quad \bar{y} = \frac{\int y dW}{W} \quad \bar{z} = \frac{\int z dW}{W}$$

Determination of CG

Substituting *W* = *mg* and *dW* = *gdm*

→ $$\bar{x} = \frac{\int x dm}{m} \quad \bar{y} = \frac{\int y dm}{m} \quad \bar{z} = \frac{\int z dm}{m}$$

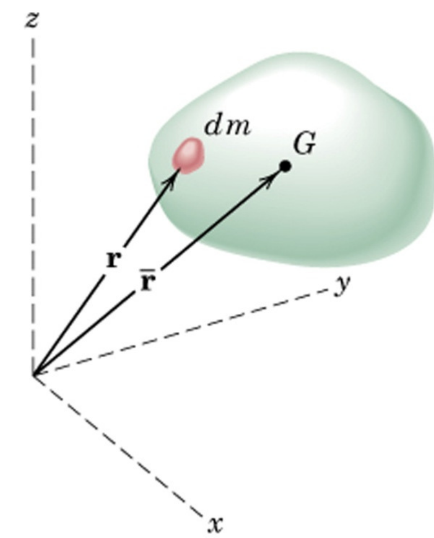
In vector notations:

Position vector for elemental mass: $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$

Position vector for mass center G: $\bar{\mathbf{r}} = \bar{x}\mathbf{i} + \bar{y}\mathbf{j} + \bar{z}\mathbf{k}$

→ $$\bar{\mathbf{r}} = \frac{\int \mathbf{r} dm}{m}$$

The above equations are the components of this single vector equation



Density ***ρ*** of a body = **mass per unit volume**

→ Mass of a differential element of volume *dV* → *dm* = *ρdV*

→ *ρ* may not be constant throughout the body

$$\bar{x} = \frac{\int x\rho dV}{\int \rho dV} \quad \bar{y} = \frac{\int y\rho dV}{\int \rho dV} \quad \bar{z} = \frac{\int z\rho dV}{\int \rho dV}$$

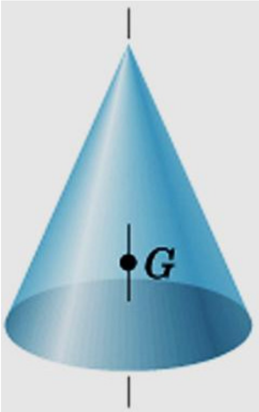
Center of Mass and Centroids

Center of Mass: Following equations independent of *g*

$$\bar{x} = \frac{\int x\,dm}{m} \qquad \bar{y} = \frac{\int y\,dm}{m} \qquad \bar{z} = \frac{\int z\,dm}{m} \qquad \bar{\mathbf{r}} = \frac{\int \mathbf{r}\,dm}{m} \qquad \bar{x} = \frac{\int x\rho\,dV}{\int \rho\,dV} \qquad \bar{y} = \frac{\int y\rho\,dV}{\int \rho\,dV} \qquad \bar{z} = \frac{\int z\rho\,dV}{\int \rho\,dV}$$

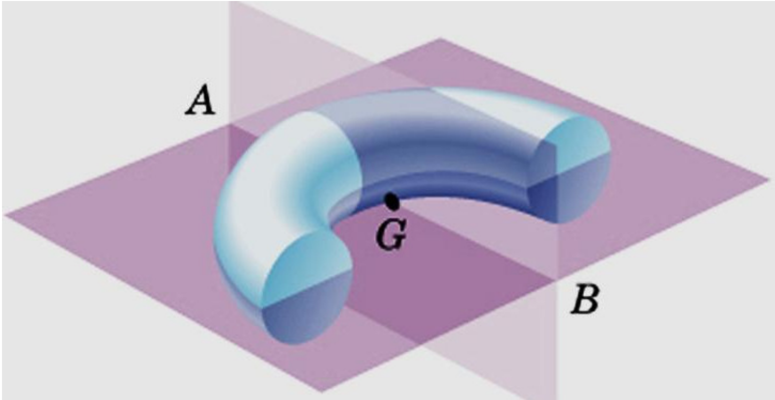
→ They define a unique point, which is a function of distribution of mass
→ This point is Center of Mass (CM)
→ CM coincides with CG as long as gravity field is treated as uniform and parallel
→ CG or CM may lie outside the body

CM always lie on a line or a plane of symmetry in a homogeneous body



Right Circular Cone
CM on central axis

Half Right Circular Cone
CM on vertical plane of symmetry



Half Ring
CM on intersection of two planes of symmetry (line AB)